Mj, Mahan

Cannon-Thurston maps for surface groups. (English) Zbl 1301.57013

Ann. Math. (2) 179, No. 1, 1-80 (2014).

In a preprint from 1985, Cannon and Thurston raise the following question: Suppose that a surface group $\pi_1(S)$ acts freely and properly discontinuously on hyperbolic 3-space $\mathbb{H}^3$ by isometries such that the quotient manifold has no accidental parabolics. Does the inclusion of the universal covering of S into $\mathbb{H}^3$ extend continuously to the boundary (the compactifications of universal covers)? Such extensions became known as Cannon-Thurston maps (see also the paper of *J. W. Cannon* and *W. P. Thurston* [Geom. Topol. 11, 1315–1355 (2007; Zbl 1136.57009)]). Floyd had proved this for geometrically finite Kleinian groups [*W. J. Floyd*, Invent. Math. 57, 205–218 (1980; Zbl 0428.20022)], Cannon and Thurston proved it for fibers of closed hyperbolic 3-manifolds fibering over the circle (the case of a doubly degenerate Kleinian surface group where the extension gives a sphere-filling or Peano curve; they also pointed out that for a simply degenerate Kleinian surface group this is equivalent, via the Carathéodory extension theorem, to asking if the limit set is locally connected). In the present substantial and involved paper, this is proved in general, for representations of surface groups into $\mathrm{PSL}_2(\mathbb{C})$ without accidental parabolics; in particular, the set of limit points of the image of the representation is locally connected. As a consequence, the author proves in general that connected limit sets of finitely generated Kleinian groups are locally connected (as he notes, it is a classical fact from general topology that a compact, connected, locally connected metric space is homeomorphic to a Peano continuum, i.e. the continuous image of a closed interval; hence, asking if the limit set is locally connected is equivalent to asking if there is some parametrization by a closed interval). The complexity of the proofs is reflected also by the fact that a first version of the paper was submitted already in 2006. An analogous question for general Kleinian groups is stated as problem 14 in Thurston's famous and most influential problem list on hyperbolic 3-manifolds and Kleinian groups [*W. P. Thurston*, Bull. Am. Math. Soc., New Ser. 6, 357–379 (1982; Zbl 0496.57005)]; extending the techniques of the present paper, the author offers a positive answer also for this more general case in a preprint [Cannon-Thurston Maps for Kleinian Groups, arXiv1002.0996].

The proofs of the present paper use the Minsky model for surface groups, crucial also for the recent solution of Thurston's ending lamination conjecture due to *Y. Minsky* [Ann. Math. (2) 171, No. 1, 1–107 (2010; Zbl 1193.30063)] and Brock-Canary-Minsky [*J. F. Brock* et al., Ann. Math. (2) 176, No. 1, 1–149 (2012; Zbl 1253.57009)] (roughly stating that the isometry type of a hyperbolic 3-manifold uniformized by a f.g. Kleinian group is determined by its end invariants).

Reviewer: Bruno Zimmermann (Trieste)

MSC:

57M50 General geometric structures on low-dimensional manifolds
30F40 Kleinian groups (aspects of compact Riemann surfaces and uniformization)

Keywords:

Cannon-Thurston map; Kleinian group; limit set; local connectivity; Peano curve

Full Text: DOI arXiv

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