Wang, Jianfeng; **Huang, Qiong Xiang**; **Teo, Kee L.**; **Belardo, Francesco**; **Liu, Ru Ying**; **Ye, Cheng Fu**

Almost every complement of a tadpole graph is not chromatically unique. (English)

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Summary: The study of chromatically unique graphs has been drawing much attention and many results are surveyed in [*F. M. Dong* et al., Chromatic polynomials and chromaticity of graphs. Singapore: World Scientific Publishing (2005; Zbl 1070.05038); *K. M. Koh* and *K. L. Teo*, Graphs Comb. 6, No. 3, 259–285 (1990; Zbl 0727.05023); Discrete Math. 172, No. 1–3, 59–78 (1997; Zbl 0879.05031)]. The notion of adjoint polynomials of graphs was first introduced and applied to the study of the chromaticity of the complements of the graphs by *R. Y. Liu* [Discrete Math. 172, No. 1–3, 85–92 (1997; Zbl 0878.05030)] (see also [Dong et al., loc. cit.]). Two invariants for adjoint equivalent graphs that have been employed successfully to determine chromatic unique graphs were introduced by Liu [loc. cit.] and by Dong et al. [loc. cit.], respectively. In this paper, we shall utilize, among other things, these two invariants to investigate the chromaticity of the complement of the tadpole graphs $C_n(P_m)$, the graph obtained from a path P_m and a cycle C_n by identifying a pendant vertex of the path with a vertex of the cycle. Let G stand for the complement of a graph G. We prove the following results:

1. The graph $\overline{C_{n-1}(P_2)}$ is chromatically unique if and only if $n \neq 5, 7$.
2. Almost every $\overline{C_n(P_m)}$ is not chromatically unique, where $n \geq 4$ and $m \geq 2$.

MSC:

05C15 Coloring of graphs and hypergraphs
05C60 Isomorphism problems in graph theory (reconstruction conjecture, etc.) and homomorphisms (subgraph embedding, etc.)

Keywords:

chromatic polynomials; chromatically unique; adjoint polynomials; adjointly unique graphs