

Hayashi, Masafumi; Kohatsu-Higa, Arturo; Yûki, Gô

**Local Hölder continuity property of the densities of solutions of SDEs with singular coefficients.** (English) [Zbl 1294.60081](#)

J. Theor. Probab. 26, No. 4, 1117-1134 (2013).

The authors consider a homogeneous Itô process

$$X_t = x + \int_0^t \sigma(X_s) dB_s + \int_0^t b(X_s) ds, \quad 0 \leq t \leq T,$$

assuming that:

- $B$  is a real Brownian motion;
- $\sigma$  and  $b$  are Borelian bounded on  $I := ]y - r, y + r[$ ;
- $\sigma \in C_b^\infty(I)$  and  $\sigma \geq \sigma_0 > 0$  on  $I$ ; and
- $b/\sigma$  is an  $\alpha$ -Hölderian on  $I$ , for some  $0 < \alpha < 1$ .

Then the authors prove that for any  $0 < \gamma < \alpha$  and  $0 \leq t \leq T$ ,  $X_t$  admits a  $\gamma$ -Hölderian density on  $]y - \frac{r}{6}, y + \frac{r}{6}[$ . They proceed by a Girsanov transform supporting the drift term, localization to  $I$ , smoothing of the Girsanov exponential martingale, and Malliavin calculus.

Reviewer: Jacques Franchi (Strasbourg)

#### MSC:

**60H07** Stochastic calculus of variations and the Malliavin calculus

Cited in **9** Documents

**60H10** Stochastic ordinary differential equations (aspects of stochastic analysis)

#### Keywords:

Malliavin calculus; non-smooth drift; density function

**Full Text:** [DOI](#)

#### References:

- [1] Cherny, A., Engelbert, H.: Singular Stochastic Differential Equations. Lecture Notes in Math., vol. 1858. Springer, Berlin (2005) · [Zbl 1071.60003](#)
- [2] Fournier, N.; Printems, J., Absolute continuity for some one dimensional processes, Bernoulli, 16, 343-360, (2010) · [Zbl 1248.60062](#)
- [3] Friedman, A.: Partial Differential Equations of Parabolic Type. Prentice-Hall, New York (1964) · [Zbl 0144.34903](#)
- [4] Itô, K., McKean, H.P. Jr.: Diffusion Processes and Their Sample Paths, 2nd edn. Springer, Berlin (1974) · [Zbl 0285.60063](#)
- [5] Kohatsu-Higa, A., Tanaka, A.: A Malliavin Calculus method to study densities of additive functionals of SDE's with irregular drifts. Ann. Inst. Henri Poincaré (2012, to appear) · [Zbl 1248.60058](#)
- [6] Krylov, N.V.: Lectures on Elliptic and Parabolic Equations in Sobolev Spaces. Graduate Studies in Mathematics, vol. 96. AMS, Providence (2008) · [Zbl 1147.35001](#)
- [7] Kusuoka, S., Existence of densities of solutions of stochastic differential equations by Malliavin calculus, J. Funct. Anal., 258, 758-784, (2010) · [Zbl 1198.60025](#)
- [8] Ladyzenskaja, O.A., Solonnikov, V.A., Ural'ceva, N.N.: Linear and Quasi-Linear Equations of Parabolic Type. Translations of Mathematical Monographs, vol. 23. AMS, Providence (1968)
- [9] Marco, S.D., Smoothness and asymptotic estimates of densities for SDEs with locally smooth coefficients and applications to square root-type diffusions, Ann. Appl. Probab., 21, 1282-1321, (2011) · [Zbl 1246.60082](#)
- [10] Marco, S.D.: On probability distributions of diffusions and financial models with non-globally smooth coefficients. Ph.D. Thesis, <http://cermics.enpc.fr/~de-marcs/home.html> · [Zbl 1246.60082](#)
- [11] Nualart, D.: The Malliavin Calculus and Related Topics, 2nd edn. Springer, Berlin (2006) · [Zbl 1099.60003](#)

[12] Williams, D.: Probability with Martingales. Cambridge University Press, Cambridge (1991) · [Zbl 0722.60001](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.