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Characterization and recognition of d.c. functions. (English) Zbl 1284.26014

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Summary: A function $f:\Omega\to\mathbb{R}$, where Ω is a convex subset of the linear space X, is said to be d.c. (difference of convex) if $f=g-h$ with $g,h:\Omega\to\mathbb{R}$ convex functions. While d.c. functions find various applications, especially in optimization, the problem to characterize them is not trivial. There exist a few known characterizations involving cyclically monotone set-valued functions. However, since it is not an easy task to check that a given set-valued function is cyclically monotone, simpler characterizations are desired. The guideline characterization in this paper is relatively simple (Theorem 2.1), but useful in various applications. For example, we use it to prove that piecewise affine functions in an arbitrary linear space are d.c. Additionally, we give new proofs to the known results that $C^{1,1}$ functions and lower-C^2 functions are d.c. The main goal remains to generalize to higher dimensions a known characterization of d.c. functions in one dimension: A function $f:\Omega\to\mathbb{R}$, $\Omega\subset\mathbb{R}$ open interval, is d.c. if and only if on each compact interval in Ω the function f is absolutely continuous and has a derivative of bounded variation. We obtain a new necessary condition in this direction (Theorem 3.8). We prove an analogous sufficient condition under stronger hypotheses (Theorem 3.11). The proof is based again on the guideline characterization. Finally, we obtain results concerning the characterization of convex and d.c. functions obeying some kind of symmetry.

MSC:

26B25 Convexity of real functions of several variables, generalizations
49J52 Nonsmooth analysis

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generalized convexity; d.c. functions; characterization of d.c. functions

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References:

[1] Alexandroff, A.D., Almost everywhere existence of the second differential of a convex function and some properties of convex surfaces connected with it, Ucenye Zapiski Leningr. Gos. Univ. Ser. Mat., 37, 3-35, (1939)

[2] Blanquero, R.; Carrizosa, E., Optimization of the norm of a vector-valued DC function and applications, J. Optim. Theory Appl., 107, 245-260, (2000) · Zbl 1064.90034

[3] Bougeard, M.L., Morse theory for some lower-C\^{}\{2\} functions in finite dimension, Math. Program., 41, 141-159, (1988) · Zbl 0646.49029

[4] Bui, A.; Bui, M.; Tuy, H., A nonconvex optimization problem arising from distributed computing, Mathematica, 43, 151-165, (2001) · Zbl 1097.90565

[5] Conway J.B.: A Course in Functional Analysis. Graduate Texts in Mathematics, vol. 96. Springer, New York (1990) · Zbl 0706.46003

[6] Dempe, S.; Gadhi, N., Necessary optimality conditions of a D.C. set-valued bilevel optimization problem, Optimization, 57, 777-793, (2008) · Zbl 1152.90587

[7] Demyanov, V.; Rubinov, A.; Demyanov, V. (ed.); Rubinov, A. (ed.), An introduction to quasidifferentiable calculus, 1-30, (2000), Dordrecht · Zbl 0949.00047

[8] Duda, J.; Veselý, L.; Zajíček, L., On d.c. functions and mappings, Atti Sem. Mat. Fis. Univ. Modena, 51, 111-138, (2003) · Zbl 1072.46025

[9] Elhilali, A.A., Caractérisation des fonctions DC, Ann. Sci. Math. Québec, 20, 1-13, (1996) · Zbl 0915.49014

[10] Elhilali, A.A.; Lafhim, L.; Metrane, A., Fonctions DC vectorielles, Math. Rech. Appl., 6, 37-55, (2004) · Zbl 1197.49010

[11] Ellaia, R.: Contribution à à l'Analyse et l'Optimisation de Differences de Fonctions Convexes. Ph.D. Thesis, l'Université Paul Sabatier (1984)

[12] Ellaia, R.; Hassouni, A., Characterization of nonsmooth functions through their generalized gradients, Optimization, 22, 401-416, (1991) · Zbl 0734.49005

[13] Ginchev, I.; Martínez-Legaz, J.E., Characterization of d.c. functions in terms of quasidifferentials, Nonlinear Anal., 74, 6781-6787, (2011) · Zbl 1229.90137

[14] Hartman, P., On functions representable as a difference of convex functions, Pac. J. Math., 9, 707-713, (1959) · Zbl 0093.06401

[15] Hiriart-Urruty, J.-B.: Generalized Differentiability, Duality and Optimization for Problems Dealing with Differences of Convex Functions. Convexity and Duality in Optimization (Groningen, 1984), Lecture Notes in Economics and Mathematical Systems, vol. 256, pp. 37-70. Springer, Berlin (1985)

[16] Laghdir, M., Optimality conditions in DC-constrained optimization, Acta Math. Vietnam., 30, 169-179, (2005) · Zbl 1155.90474

[17] Lahoussine, L.; Elhilali, A.A.; Gadhi, N., Set-valued mapping monotonicity as characterization of D.C. functions, Positivity, 13, 399-405, (2009) · Zbl 1222.46005

[18] Martínez-Legaz, J.-E., Volle, M.: Duality for d.c. optimization over compact sets. Optimization Theory (Mátraháza, 1999), Appl. Optim., vol. 59, pp. 139-146. Kluwer Academic Publishers, Dordrecht (2001) · Zbl 1041.90067

[19] Moudafi, A., Convergence of a proximal-type method for DC functions, J. Appl. Funct. Anal., 1, 285-291, (2006) · Zbl 1137.49017

[20] Moudafi, A.; Maingé, P.-E., On the convergence of an approximate proximal method for DC functions, J. Comput. Math., 24, 475-480, (2006) · Zbl 1104.65058

[21] Penot, J.P.; Bougeard, M.L., Approximation and decomposition properties of some classes of locally D.C. functions, Math. Program., 41, 195-227, (1988) · Zbl 0666.49005

[22] Rockafellar R.T.: Convex Analysis. Princeton Mathematical Series, No. 28. Princeton University Press, Princeton (1970) · Zbl 0193.18401

[23] Sun, W.-Y.; Sampaio, R.J.B.; Candido, M.A.B., Proximal point algorithm for minimization of DC function, J. Comput. Math., 21, 451-462, (2003) · Zbl 1107.90427

[24] Volle, M., Duality principles for optimization problems dealing with the difference of vector-valued convex mappings, J. Optim. Theory Appl., 114, 223-241, (2002) · Zbl 1041.90068

[25] Yomdin, Y., On functions representable as a supremum of a family of smooth functions, SIAM J. Math. Anal., 14, 239-246, (1983) · Zbl 0524.26010

[26] Zălinescu C.: Convex Analysis in General Vector Spaces. World Scientific Publishing Co., Inc., River Edge (2002) · Zbl 1023.46003

[27] Zelený, M., An example of a C\^{}\{1,1\} function, which is not a d.c. function, Comment. Math. Univ. Carolin., 43, 149-154, (2002) · Zbl 1090.46012

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