

**Seveso, Marco Adamo**

**The Teitelbaum conjecture in the indefinite setting.** (English) Zbl 1288.11051  
*Am. J. Math.* 135, No. 6, 1525-1557 (2013).

Let  $f$  be a new form of level  $N$  and even weight  $k + 2 \geq 2$ . Assume  $N = pN^+N^-$  where the factors are prime to each other, and  $N^-$  is square free. There is an associated invariant  $\mathcal{L}^{N^-}(f)$  attached to this factorization. In the setting of  $p$ -adic  $L$ -functions associated to  $f$  by Mazur-Tate-Teitelbaum, there are  $\mathcal{L}$ -invariants associated to  $f$ :  $\mathcal{L}_C(f)$  by *R. F. Coleman* [*Contemp. Math.* 165, 21–51 (1994; [Zbl 0838.11033](#))],  $\mathcal{L}_{FM}(f)$  by Fontaine and *B. Mazur* [*Contemp. Math.* 165, 1–20 (1994; [Zbl 0846.11039](#))],  $\mathcal{L}_B(f)$  by *Ch. Breuil* [*Astérisque* 331, 65–115 (2010; [Zbl 1246.11106](#))]. This paper proves that when  $N^-$  has an even number of prime factors (thus the quaternion algebra ramified over all primes dividing  $N^-$  is indefinite),

$$\mathcal{L}^{N^-}(f) = -2(\log a_p)'(k).$$

Here one associates a Hida family to  $f$  such that for  $n \geq k$  integer, there is  $f^n := \sum_{i \geq 1} a_i(n)q^i$  modular form of weight  $n + 2$  and level  $N$  in a neighborhood of  $k$  (under  $p$ -adic topology). The derivative is  $\frac{d}{dn}(\log a_p(n))|_{n=k}$ .

The author shows that  $\mathcal{L}^{N^-}(f)$  equals the other invariants  $\mathcal{L}_C(f)$ ,  $\mathcal{L}_{FM}(f)$  and  $\mathcal{L}_B(f)$ , and also is independent of the choice of  $N^-$  in the factorization of  $N$ . When  $N^-$  has odd number of prime factors (thus the corresponding quaternion algebra is definite), the results are known (at least when  $k = 0$ ) thanks to the work of *M. Bertolini*, *H. Darmon* and *A. Iovita* [*Astérisque* 331, 65–115 (2010; [Zbl 1251.11033](#))]

Reviewer: [Zhengyu Mao \(Newark\)](#)

**MSC:**

- [11F67](#) Special values of automorphic  $L$ -series, periods of automorphic forms, cohomology, modular symbols Cited in 4 Documents
- [11F75](#) Cohomology of arithmetic groups
- [11F80](#) Galois representations

**Keywords:**

*L*-invariants of modular forms;  $p$ -adic  $L$ -function; Darmon cycle

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