Kocel-Cynk, Beata; **Pawłucki, Wiesław**; **Valette, Anna**

A short geometric proof that Hausdorff limits are definable in any o-minimal structure. (English) Zbl 1309.14046

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Suppose that $\mathcal{A}$ is an o-minimal expansion of the real field. If $\mathcal{A}$ is an $\mathcal{R}$-definable family of nonempty compact subsets of $\mathbb{R}^n$ then the closure of $\mathcal{A}$ in the Hausdorff metric is also a definable family. This theorem has several proofs. The original proof by *D. Marker* and *C. I. Steinhorn* [J. Symb. Log. 59, No. 1, 185–198 (1994; Zbl 0801.03026)] used model theory. There have also been several other model-theoretic proofs. *J. M. Lion* and *P. Speissegger* [Sel. Math., New Ser. 10, No. 3, 377–390 (2004; Zbl 1059.03031)] gave a geometric proof using blowings-up in jet spaces. Here the authors give a new geometric proof, based on the Lipschitz cell decompositions introduced by the second and first authors (see for instance [Ill. J. Math. 52, No. 3, 1045–1063 (2008; Zbl 1222.32019)]).

Reviewer: Gareth Jones (Manchester)

MSC:

14P10	Semialgebraic sets and related spaces
32B20	Semi-analytic sets, subanalytic sets, and generalizations
03C64	Model theory of ordered structures; o-minimality
14P15	Real-analytic and semi-analytic sets

Keywords:

Hausdorff limit; o-minimal structure

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