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**Global regularity for the two-dimensional anisotropic Boussinesq equations with vertical dissipation.** (English) [Zbl 1284.35140](#)

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This paper establishes the global in time existence of classical solutions to the two-dimensional anisotropic Boussinesq equations with vertical dissipation. When only vertical dissipation is present, there is no direct control on the horizontal derivatives and the global regularity problem is very challenging. To solve this problem, the authors bound the derivatives in terms of the  $L^\infty$ -norm of the vertical velocity  $v$  and prove that  $\|v\|_{L^r}$  with  $2 \leq r < \infty$  does not grow faster than  $\sqrt{r \log r}$  at any time as  $r$  increases. A delicate interpolation inequality connecting  $\|v\|_{L^\infty}$  and  $\|v\|_{L^r}$  then yields the desired global regularity.

Reviewer: [Titus Petrila \(Cluj-Napoca\)](#)

**MSC:**

[35G35](#) Systems of linear higher-order PDEs

[76D05](#) Navier-Stokes equations for incompressible viscous fluids

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**Keywords:**

two-dimensional anisotropic Boussinesq equations; vertical dissipation; global regularity of classical solutions

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