Blaszczyszyn, Bartlomiej; Yogeshwaran, Dhandapani

Clustering and percolation of point processes. (English) Zbl 1291.60099

Electron. J. Probab. 18, Paper No. 72, 20 p. (2013).

Let Φ be a point process in $\mathbb{R}^d$, the d-dimensional Euclidean space. For $r \geq 0$ and $k \geq 1$, the coverage number field is $V_{\Phi,r}(y) = \sum_{x_i \in \Phi} I[y \in B_r(x_i)]$, where $B_r(x)$ denotes the Euclidean ball of radius r centered at x, and $I[A]$ is the indicator function of A. The k-covered set is defined as $C_k(\Phi, r) = \{y : V_{\Phi,r}(y) \geq k\}$. Define the critical radius for k-percolation as $r_C^k(\Phi) = \inf\{r : \mathrm{P}(C_k(\Phi, r) \text{ percolates}) > 0\}$, where percolation means existence of an unbounded connected subset. Note that $C_1(\Phi, r)$ is the standard Boolean model or continuum percolation model. The authors show that point processes having voids probabilities and moment measures smaller than a homogeneous Poisson point process (weakly sub-Poisson) exhibit a nontrivial phase transition in the percolation of their level-set coverage models. The authors develop more general methods suitable for the study of percolation of level sets of additive shot-noise fields. Besides $C_k(\Phi, r)$, the authors apply these methods to study percolation of coverage by a signal-to-interference-and-noise ratio, see [*O. Dousse* et al., J. Appl. Probab. 43, No. 2, 552–562 (2006; Zbl 1154.82311)] in the non-Poisson setting. The rest of the paper is organized as follows. The necessary notions, notations as well as some preliminary results are introduced and recalled in Section 2. In Section 3, the authors state and prove the main results regarding the existence of the phase transition for percolation models driven by sub-Poisson point processes. In Section 4, a Cox point process, which is "more clustered" than the Poisson point process and whose Boolean model percolates for arbitrary small radius, is constructed.

Reviewer: Viktor Ohanyan (Erevan)

MSC:

60G55	Point processes (e.g., Poisson, Cox, Hawkes processes)
82B43	Percolation
60K35	Interacting random processes; statistical mechanics type models; percolation theory
60D05	Geometric probability and stochastic geometry

Keywords:
point process; Boolean model; percolation; phase transition; shot-noise field; perturbed lattices

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