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Turing degrees of multidimensional SFTs. (English) Zbl 1417.03241
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Summary: In this paper, we are interested in computability aspects of subshifts and in particular Turing degrees of two-dimensional subshifts of finite type (SFTs) (i.e., tilings). To be more precise, we prove that, given any Π_1^0 class P of $\{0, 1\}^{\mathbb{N}}$, there is an SFT X such that $P \times \mathbb{Z}^2$ is recursively homeomorphic to $X \setminus U$, where U is a computable set of points. As a consequence, if P contains a computable member, P and X have the exact same set of Turing degrees. On the other hand, we prove that, if X contains only non-computable members, some of its members always have different but comparable degrees. This gives a fairly complete study of Turing degrees of SFTs.

MSC:

03D28 Other Turing degree structures
37B10 Symbolic dynamics
52C20 Tilings in 2 dimensions (aspects of discrete geometry)

Cited in 8 Documents

Keywords:

tilings; subshift of finite type; undecidability; Π_1^0 classes; Turing degree

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