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Uniform hypergraphs containing no grids. (English) Zbl 1278.05161

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Summary: A hypergraph is called an $r\times r$ grid if it is isomorphic to a pattern of r horizontal and r vertical lines, i.e., a family of sets $\{A_1,\dots,A_r,B_1,\dots,B_r\}$ such that $A_i\cap A_j=B_i\cap B_j=\emptyset$ for $1\le i<j\le r$ and $|A_i\cap B_j|=1$ for $1\le i,j\le r$. Three sets C_1,C_2,C_3 form a *triangle* if they pairwise intersect in three distinct singletons, $|C_1\cap C_2|=|C_2\cap C_3|=|C_3\cap C_1|=1$, $C_1\cap C_2\neq C_1\cap C_3$. A hypergraph is linear, if $|E\cap F|\le 1$ holds for every pair of edges $E\neq F$. In this paper we construct large linear r-hypergraphs which contain no grids. Moreover, a similar construction gives large linear r-hypergraphs which contain neither grids nor triangles. For $r\ge 4$ our constructions are almost optimal. These investigations are motivated by coding theory: we get new bounds for optimal superimposed codes and designs.

MSC:

05C65 Hypergraphs
05C42 Density (toughness, etc.)
05D05 Extremal set theory
11B25 Arithmetic progressions

Keywords:

Turán hypergraph problem; density problems; union free hypergraphs; superimposed codes

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