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Quasi-isometric maps and Floyd boundaries of relatively hyperbolic groups. (English)

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This paper is a part of a larger program initiated by *V. Gerasimov* [Geom. Funct. Anal. 19, No. 1, 137-169 (2009; Zbl 1226.20037)].

Let a discrete finitely generated group G act by homeomorphisms of a compactum (compact Hausdorff space) T in a way that the action is properly discontinuous on triples and cocompact on pairs. The first main result of the paper is a characterization of the preimages under the Floyd map of parabolic points in T. From this theorem together with their previous results the authors deduce a complete description of the Floyd map for relatively hyperbolic groups. In the second theorem they use the Floyd completion to prove that the property of relative hyperbolicity is invariant under quasi-isometric maps. This result is then generalized to α-isometric maps with any polynomial distortion function α.

One of the main technical ingredients of the proofs is a generalization of *A. Karlsson*'s lemma saying that the Floyd length of any (quasi)geodesic in the Cayley graph of a finitely generated group situated far away from the origin is small [Commun. Algebra 31, No. 11, 5361-5376 (2003; Zbl 1036.20032)].

As an application of their method, in the Appendix the authors give a short proof of Bowditch's Theorem characterizing hyperbolicity [*B. H. Bowditch*, J. Am. Math. Soc. 11, No. 3, 643-667 (1998; Zbl 0906.20022)].

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MSC:

20F67 Hyperbolic groups and nonpositively curved groups
20F65 Geometric group theory
57M07 Topological methods in group theory
20F05 Generators, relations, and presentations of groups

Keywords:

relatively hyperbolic groups; Floyd boundaries; convergence actions; quasi-isometric maps; finitely generated groups; actions by homeomorphisms

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