zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Bowditch, B. H.

Stacks of hyperbolic spaces and ends of 3-manifolds. (English) Zbl 1297.57044

Hodgson, Craig D. (ed.) et al., Geometry and topology down under. A conference in honour of Hyam Rubinstein, Melbourne, Australia, July 11–22, 2011. Proceedings. Providence, RI: American Mathematical Society (AMS) (ISBN 978-0-8218-8480-5/pbk). Contemporary Mathematics 597, 65-138 (2013).

The paper under review discusses an alternative approach to the construction of Cannon-Thurston maps and to the proof of the ending lamination theorem of Brock-Canary-Minsky

The general framework of this paper is given by the notion of a "hyperbolic stack". Here a stack is a geodesic space Ξ together with a sequence of pairwise uniformly quasi-isometric spaces X_i and embeddings $f_i : X_i \to \Xi$ that are uniformly straight (distances in Ξ are bounded above and below in terms of distances in X_i), whose union is quasi-dense in Ξ, and such that the Hausdorff-distances between the images of consecutive X_i are bounded above while the distance between X_i and X_j is bounded away from zero by a linear function of $\mid i - j \mid$. A stack can be constructed from a sequence of uniformly quasi-isometric spaces and it is unique up to quasi-isometry.

A hyperbolic stack is a stack, where Ξ and X_i are uniformly hyperbolic spaces. (One may think of surfaces $\widetilde{\Sigma} \times \{i\} \subset \widetilde{\Sigma} \times \mathbb{R}$ in the universal covering of a hyperbolic 3-manifold $\Sigma \times \mathbb{R}$.) Since there is a well-defined (i.e., up to bounded distance) quasi-isometry $X_i \to X_0$, the Gromov-boundaries of all X_i have a canonical homeomorphism to $\partial^0\Xi := \partial X_0$. Section 2 of the paper under review constructs a "Cannon-Thurston map" $\partial^0\Xi \to \partial\Xi$ such that $X_0 \cup \partial X_0 \to \Xi \cup \partial\Xi$ is continuous. It is proved that $\partial\Xi$ is a metrisable Peano continuum.

When Ξ is decomposed into semi-infinite stacks $\Xi = \Xi^+ \cup \Xi^-$ with $\Xi^+ \cap \Xi^- = X_0$, then $\Xi^\pm$ are hyperbolic, $\partial^0\Xi \to \partial\Xi$ factors over $\partial\Xi^\pm$ and $\partial\Xi^\pm$ is a dendrite, i.e. every pair of points is separated by a cut point. If all X_i are hyperbolic planes, then the Cannon-Thurston map $\partial^0\Xi \to \partial\Xi$ is proven to be surjective and the maps $\partial^0\Xi \to \partial\Xi^\pm$ are the quotient maps for the equivalence relations given by unique laminations $\Lambda^\pm$ of ∂H^2.

These laminations are called the ending laminations of the stack and Section 3 of the paper under review is devoted to their study in the case that the X_i are universal covers of hyperbolic surfaces. It is shown that any ending lamination of a hyperbolic surface stack is regular, i.e. there is a linear function, f, such that for any interval E contained in a leaf and any essential curve γ in the complement of E one has $length(E) \leq f(length(\gamma))$. It is shown that regular laminations have a transverse measure, unique up to scaling.

Section 4 gives an independent proof of a result of *L. Mosher* [Geom. Topol. 7, 33–90 (2003; Zbl 1021.57009)]: a path β in the thick part of Teichmüller space is bounded distance from a Teichmüller geodesic if and only if the universal covering of $P(\beta)$ is Gromov hyperbolic. Here $P(\beta)$ means the (up to bilipschitz equivalence canonical) Riemannian manifold $\Sigma \times \mathbb{R}$ such that $\Sigma \times \{t\}$ is uniformly bilipschitz equivalent to the hyperbolic metric $\beta(t)$. The universal covering of $P(\beta)$ is equivariantly quasi-isometric to the stack Ξ build from the $X_i = \widetilde{\Sigma} \times \{i\}$, so that this result fits in the above-mentioned more general setting. In general, a stack turns out to be hyperbolic if the functions $i \to d_i(x_i, y_i)$ are uniformly quasiconvex for all chains $x_i, y_i \in X_i$.

In the final Section 4.9 the author discusses how this can be applied to prove the Ending Lamination Theorem in the case of positive injectivity radius.

For the entire collection see [Zbl 1272.57002].

Reviewer: Thilo Kuessner (Seoul)

MSC:

57M60 Group actions on manifolds and cell complexes in low dimensions
32G15 Moduli of Riemann surfaces, Teichmüller theory (complex-analytic aspects in several variables)
57M50 General geometric structures on low-dimensional manifolds

Cited in **8** Documents

Keywords:

hyperbolic space; boundary; Cannon-Thurston map, Teichmüller space; 3-manifold; lamination

© 2021 FIZ Karlsruhe GmbH