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An efficient Legendre pseudospectral method for solving nonlinear quasi bang-bang optimal control problems. (English) [Zbl 1277.65104](#)

J. Appl. Math. Stat. Inform. 8, No. 2, 73-85 (2012).

Summary: In this paper, an efficient Legendre pseudospectral approach for the accurate solution of nonlinear quasi bang-bang optimal control problems (OCPs) is investigated. In this approach, after linearizing the dynamical system, control and state functions are considered as piecewise constant and piecewise continuous polynomials, respectively, and the switching points are also taken as decision variables. Furthermore, for simplicity in discretization, an integral formulation of the dynamical equations is considered. Thereby, the problem is converted into a mathematical programming problem which can be solved by well-developed parameter optimization algorithms. Through a numerical implementation we show the efficiency of the proposed method via comparing with a classical pseudospectral method and other discretization approaches.

MSC:

- 65N35 Spectral, collocation and related methods for boundary value problems involving PDEs Cited in 8 Documents
- 49J15 Existence theories for optimal control problems involving ordinary differential equations
- 90C30 Nonlinear programming
- 93C05 Linear systems in control theory
- 49M25 Discrete approximations in optimal control
- 90C05 Linear programming

Keywords:

quasi bang-bang optimal control; switching point; numerical methods; pseudospectral Legendre method

Software:

[Matlab](#)

Full Text: [DOI](#)

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