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Euler's problem and its applications in celestial mechanics and space dynamics. (English. Russian original) [Zbl 1272.70063](#)

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Summary: The numerous generalizations of the classical problem of two fixed centres are analysed, starting from the formulation of the problem and its solution by Euler in 1760 to the present day. The role played by numerous researchers in analysing this problem is noted. The publications cited indicate conclusively that the main results of generalizations of the problem and analytical and qualitative investigations had already been obtained in the nineteenth century and at the beginning of the twentieth century. Present-day researchers can only lay claim to a few occasionally productive and at the same time effective applications of individual generalizations (the Gredeaks problem, for example).

MSC:

70F15 Celestial mechanics

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