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A new characterization of convexity in free Carnot groups. (English) Zbl 1272.31009
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Summary: A characterization of convex functions in \mathbb{R}^N states that an upper semicontinuous function u is convex if and only if $u(Ax)$ is subharmonic (with respect to the usual Laplace operator) for every symmetric positive definite matrix A . The aim of this paper is to prove that an analogue of this result holds for free Carnot groups \mathbb{G} when considering convexity in the viscosity sense. In the subelliptic context of Carnot groups, the linear maps $x \mapsto Ax$ of the Euclidean case must be replaced by suitable group isomorphisms $x \mapsto T_A(x)$, whose differential preserves the first layer of the stratification of $\text{Lie}(\mathbb{G})$.

MSC:

31C05 Harmonic, subharmonic, superharmonic functions on other spaces
26B25 Convexity of real functions of several variables, generalizations
43A80 Analysis on other specific Lie groups
35J70 Degenerate elliptic equations

Cited in **2** Documents

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