Bauer, Thomas; Harbourne, Brian; Knutsen, Andreas Leopold; Küronya, Alex; Müller-Stach, Stefan; Roulleau, Xavier; Szemberg, Tomasz

Negative curves on algebraic surfaces. (English) Zbl 1272.14009

Duke Math. J. 162, No. 10, 1877-1894 (2013).

Let X be a smooth complex projective surface. A classical but nevertheless completely open conjecture claims that there exists a positive integer $b(X)$ such that for every integral curve $C \subset X$, the self-intersection number C^2 is bounded below by $-b(X)$. The goal of this paper is to give some evidence in favour of this conjecture: a natural strategy for a counterexample is to consider a surface X admitting an endomorphism, i.e., a surjective map $X \to X$ of degree at least two, and study the pull-back of a curve with negative self-intersection. However the authors prove that the bounded negativity conjecture holds for every surface admitting an endomorphism. A more general strategy is to consider a surface with an interesting correspondence, for example a Hecke correspondence on a quaternionic Shimura surface of general type (we refer to the paper for precise definitions). Hecke correspondence are known to preserve the set of Shimura curves on the surface, yet again the authors prove that Shimura curves verify the bounded negativity conjecture. Finally the authors prove a weak form of the conjecture if X is a non-uniruled surface: let X be surface of non-negative Kodaira dimension. Then there exists a positive integer $b(X, g)$ such that $C^2 \geq -b(X, g)$ for every integral curve $C \subset X$ of geometric genus at most g.

Reviewer: Andreas Höring (Nice)

MSC:

14C17	Intersection theory, characteristic classes, intersection multiplicities in algebraic geometry
14C20	Divisors, linear systems, invertible sheaves
14G35	Modular and Shimura varieties

Keywords:
negative curves; bounded negativity; Miyaoka-Yau inequality

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References:

[1] W. L. Baily and A. Borel, Compactification of arithmetic quotients of bounded symmetric domains , Ann. of Math. (2) 84 (1966), 442-528. · Zbl 0154.08602 · doi:10.2307/1970457

[2] W. P. Barth, K. Hulek, C. A. M. Peters, and A. Van de Ven, Compact Complex Surfaces , 2nd ed., Ergeb. Math. Grenzgeb. (3) 4 , Springer, Berlin, 2004. · Zbl 1036.14016

[3] W. Barth and I. Nieto, Abelian surfaces of type \((1,3)\) and quartic surfaces with \(16\) skew lines , J. Algebraic Geometry 3 (1994), 173-222. · Zbl 0809.14027

[4] G. Barthel, F. Hirzebruch, and T. Höfer, Geradenkonfigurationen und Algebraische Flächen , Aspects Math. D4 , Vieweg & Sohn, Braunschweig, 1987.

[5] T. Bauer, Smooth Kummer surfaces in projective three-space , Proc. Amer. Math. Soc. 125 (1997), 2537-2541. · Zbl 0883.14019 · doi:10.1090/S0002-9939-97-04089-6

[6] T. Bauer, C. Bocci, S. Cooper, S. Di Rocco, M. Dumnicki, B. Harbourne, K. Jabbusch, A. L. Knutsen, A. Küronya, R. Miranda, J. Roé, H. Schenck, T. Szemberg, and Z. Teitler, “Recent developments and open problems in linear series” in Contributions to Algebraic Geometry , EMS Ser. Congr. Rep., European Math. Soc., Zürich, 2012, 93-140. · Zbl 1254.14001

[7] T. Bauer and C. Schulz, Seshadri constants on the self-product of an elliptic curve , J. Algebra 320 (2008), 2981-3005. · Zbl 1171.14021 · doi:10.1016/j.jalgebra.2008.06.024

[8] A. Beauville, Complex Algebraic Surfaces , 2nd ed., London Math. Soc. Stud. Texts 34 , Cambridge Univ. Press, Cambridge, 1996. · Zbl 0849.14014

[9] D. A. Cox, The homogeneous coordinate ring of a toric variety , J. Algebraic Geom. 4 (1995), 17-50. · Zbl 0846.14032

[10] P. Deligne, “Travaux de Shimura” in Séminaire Bourbaki 1970/1971 , no. 389, Lecture Notes in Math. 244 , Springer, Berlin, 1971, 123-165.

[11] C. Fontanari, Towards bounded negativity of self-intersection on general blown-up projective planes , Comm. Algebra. 40 (2012), 1762-1765. · Zbl 1245.14012 · doi:10.1080/00927872.2011.555803

[12] Y. Fujimoto, Endomorphisms of smooth projective \(3\)-folds with non-negative Kodaira dimension , Publ. Res. Inst. Math. Sci. 38 (2002), 33-92. · Zbl 1053.14049 · doi:10.2977/prims/1145476416

[13] H. Granath, On quaternionic Shimura surfaces , Ph.D. dissertation, Chalmers University of Technology, Gothenburg, Sweden, 2002.

[14] B. Harbourne, Global aspects of the geometry of surfaces , Ann. Univ. Paedagog. Crac. Stud. Math. 11 (2010), 5-41. · Zbl 1247.14006

[15] J. Harris, The interpolation problem , workshop lecture at "Classical Algebraic Geometry Today," Mathematical Sciences Research Institute, Berkeley, California, 2009, . · jessica2.msri.org

[16] R. Hartshorne, Algebraic Geometry , Grad. Texts in Math. 52 , Springer, New York, 1977. · Zbl 0367.14001

[17] A. Hurwitz, Über Riemannsche Flächen mit gegebenen Verzweigungspunkten , Math. Ann. 103 (1891), 1-60. · Zbl 23.0429.01 · doi:10.1007/BF01199469

[18] R. Lazarsfeld, Positivity in Algebraic Geometry, I-II , Ergeb. Math. Grenzgeb. (3) 48-49 ; Springer, Berlin, 2004. · Zbl 1066.14021

[19] S.-S. Lu and Y. Miyaoka, Bounding curves in algebraic surfaces by genus and Chern numbers , Math. Res. Lett. 2 (1995), 663-676. · Zbl 0870.14020 · doi:10.4310/MRL.1995.v2.n6.a1

[20] Y. Miyaoka, The maximal number of quotient singularities on surfaces with given numerical invariants , Math. Ann. 268 (1984), 159-171. · Zbl 0521.14013 · doi:10.1007/BF01456083 · eudml:182912

[21] Y. Miyaoka, The orbibundle Miyaoka-Yau-Sakai inequality and an effective Bogomolov-McQuillan theorem , Publ. Res. Inst. Math. Sci. 44 (2008), 403-417. · Zbl 1162.14026 · doi:10.2977/prims/1210167331

[22] S. Müller-Stach, E. Viehweg, and K. Zuo, Relative proportionality for subvarieties of moduli spaces of K3 and abelian surfaces , Pure Appl. Math. Q. 5 (2009), 1161-1199. · Zbl 1222.14020 · doi:10.4310/PAMQ.2009.v5.n3.a8

[23] D. Mumford, Hirzebruch's proportionality theorem in the non-compact case , Invent. Math. 42 (1977), 239-272. · Zbl 0365.14012 · doi:10.1007/BF01389790 · eudml:142502

[24] N. Nakayama, Ruled surfaces with non-trivial surjective endomorphisms , Kyushu J. Math. 56 (2002), 433-446. · Zbl 1049.14029 · doi:10.2206/kyushujm.56.433

[25] I. H. Shavel, A class of algebraic surfaces of general type constructed from quaternion algebras , Pacific J. Math. 76 (1978), 221-245. · Zbl 0422.14022 · doi:10.2140/pjm.1978.76.221 ·