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The Taylor-Wiles method for coherent cohomology. (English) Zbl 1276.11102
J. Reine Angew. Math. 679, 125-153 (2013).

The upshot of the Taylor-Wiles method in their proof of Fermat's Last Theorem is the so-called $R = T$ theorem, where R is the deformation ring of mod p Galois representations and T is a ring of Hecke operators. This technique has been improved independently by Diamond and Fujiwara since then. It is based on a comparison of modules of automorphic cohomology over p -adic integers.

In this paper, the author shows that the Diamond-Fujiwara method can also be applied by replacing the topological cohomology by coherent cohomology of suitable automorphic vector bundles. One of the main ingredients is the works of *K.-W. Lan* and *J. Suh* [*Int. Math. Res. Not.* 2011, No. 8, 1870–1879 (2011; [Zbl 1233.11042](#))] as well as [*K.-W. Lan* and *J. Suh*, “Vanishing theorems for torsion automorphic sheaves on compact PEL-type Shimura varieties”, manuscript (2010)], which provide a vanishing theorem à la [*H. Esnault* and *E. Viehweg*, *Lectures on vanishing theorems*. DMV Seminar. 20. Basel: Birkhäuser Verlag (1992; [Zbl 0779.14003](#))] for automorphic vector bundles on Shimura varieties of PEL-type, under certain regularity and p -smallness conditions.

On the other hand, to start the Diamond-Fujiwara machine, one also need to verify the Galois hypotheses (§4.3). Results for unitary groups obtained by the French school are used; an excellent reference thereof is the Book Project [Stabilization of the trace formula, Shimura varieties, and arithmetic applications. Volume 1: On the stabilization of the trace formula. Somerville, MA: International Press (2011; [Zbl 1255.11027](#))].

As the author pointed out, although there is no new result about Galois representations, the bonus is that in the course of proving $R = T$, one obtains the freeness of $H^{q(\mathcal{F})}(\mathbb{S}_K, \mathcal{F})$ over the localized Hecke algebra. Some remarks about (i) the case of non-compact Shimura varieties (for which one might need the “interior cohomology”) and (ii) about the extension to Hida families are also given.

Reviewer: [Wen-Wei Li \(Beijing\)](#)

MSC:

[11G18](#) Arithmetic aspects of modular and Shimura varieties
[11F80](#) Galois representations

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