Haukkanen, Pentti; **Mattila, Mika**; **Merikoski, Jorma K.**; **Tossavainen, Timo**
Perpendicularity in an Abelian group. (English) Zbl 1264.51012
Int. J. Math. Math. Sci. 2013, Article ID 983607, 8 p. (2013).

Summary: We give a set of axioms to establish a perpendicularity relation in an abelian group and then study the existence of perpendicularities in $(\mathbb{Z}_n, +)$ and $(\mathbb{Q}_+, \cdot)$ and in certain other groups. Our approach provides a justification for the use of the symbol $\perp$ denoting relative primeness in number theory and extends the domain of this convention to some degree. Related to that, we also consider parallelism from an axiomatic perspective.

MSC:

51M05 Euclidean geometries (general) and generalizations
51F20 Congruence and orthogonality in metric geometry
20K01 Finite abelian groups

Full Text: DOI

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