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Embedding factorizations for 3-uniform hypergraphs. (English) Zbl 1264.05088

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Summary: In this article, two results are obtained on a hypergraph embedding problem. The proof technique is itself of interest, being the first time amalgamations have been used to address the embedding of hypergraphs. The first result finds necessary and sufficient conditions for the embedding a hyperedge-colored copy of the complete 3-uniform hypergraph of order m, K_m^3, into an r-factorization of K_n^3, providing that $n > 2m + (-1 + \sqrt{8m^2 - 16m - 7}/2$. The second result finds necessary and sufficient conditions for an embedding when not only are the colors of the hyperedges of K_m^3 given, but also the colors of all the "pieces" of hyperedges on these m vertices are prescribed (the "pieces" of hyperedges are eventually extended to hyperedges of size 3 in K_n^3 by adding new vertices to the hyperedges of size 1 and 2 during the embedding process). Both these results make progress toward settling an old question of Cameron on completing partial 1-factorizations of hypergraphs.

MSC:

05C65	Hypergraphs	Cited in **1** Review
05C60	Isomorphism problems in graph theory (reconstruction conjecture, etc.) and homomorphisms (subgraph embedding, etc.)	Cited in **2** Documents
05C70	Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)	

Keywords:

amalgamations; detachments; complete 3-uniform hypergraphs; embedding; factorization; decomposition

Full Text: DOI

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