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**Arboreal Galois representations and uniformization of polynomial dynamics.** (English)

Zbl 1280.37058

Bull. Lond. Math. Soc. 45, No. 2, 301-308 (2013).

As an analog of Tate's uniformization for elliptic curves, the author constructs an (analytic) coordinate-change near infinity for a polynomial  $f$  so that  $f$  looks like  $z^{\deg f}$  (Theorem 2). More precisely, given a monic  $f(z) \in K[z]$  over a non-Archimedean complete field such that  $\deg(f)$  is not divisible by the residue characteristic of  $K$ , there exists a Galois-equivariant invertible power series  $\Omega$  convergent in a neighborhood  $D$  near infinity such that  $\Omega(f(z)) = \Omega(z)^{\deg f}$ .

As a consequence, the author derives a dynamical analog (Theorem 1) over local fields of the open-image theorem of Serre. Assume  $[K : \mathbb{Q}_p] < \infty$ ,  $f \in K[z]$  monic of degree  $d \not\equiv 0 \pmod{p}$ , and  $P \in K$  not lying in an orbit of any critical point of  $f$ . Let  $T_{f,P}$  be the rooted  $d$ -ary tree whose nodes at level  $n$  are preimages of  $P$  by the  $n$ -th iterate  $f^n$ , with an edge between  $\alpha$  at level  $n$  and  $\beta$  at level  $n-1$  if and only if  $f(\alpha) = \beta$ . The absolute Galois group  $G_K$  acts on  $T_{f,P}$ . If  $f$  has good reduction and if  $P$  is outside the filled Julia set, the author shows that the entire  $T_{f,P}$  is inside  $D$  in terms of Theorem 2, and thus  $T_{f,P}$  is isomorphic to  $T_{z^d, \Omega(P)}$  as  $G_K$ -trees. As a result, the image of this arboreal representation is shown to have finite index inside a Kummer subgroup of  $\text{Aut}(T_{f,P})$ . It follows (Corollary 3) that for polynomials defined over number fields satisfying the conditions of Theorem 1 locally for some  $p$ , the fields of definition of preimages eventually grow by  $d$  at each iterate.

There are also other consequences of Theorem 2. Corollary 4 proves for certain polynomials a conjecture of V. Sookdeo [J. Number Theory 131, No. 7, 1229–1239 (2011; Zbl 1246.37102)], demonstrating that there are only finitely many points in any backward orbit which are integral with respect to a given nonpreperiodic point. Corollaries 5 and 6 give some uniform boundedness results for rational preimages in one-parameter families, cf. [X. Faber et al., Math. Res. Lett. 16, No. 1, 87–101 (2009; Zbl 1222.11086); A. Levin, Monatsh. Math. 168, No. 3–4, 473–501 (2012; Zbl 1302.37060)] for related results.

Reviewer: Yu Yasufuku (Tokyo)

**MSC:**

- 37P20 Dynamical systems over non-Archimedean local ground fields
- 37P05 Arithmetic and non-Archimedean dynamical systems involving polynomial and rational maps
- 11S20 Galois theory

Cited in 6 Documents

**Keywords:**

preimages of iterates; uniformization; arboreal representation; Kummer theory

**Full Text:** DOI arXiv