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Alternating direction implicit method for solving two-dimensional cubic nonlinear Schrödinger equation. (English) Zbl 1277.65073

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Summary: Four alternating direction implicit (ADI) schemes are presented for solving two-dimensional cubic nonlinear Schrödinger equations. Firstly, we give a Crank-Nicolson ADI scheme and a linearized ADI scheme both with accuracy $O(\Delta t^2+h^2)$, with the same method, use fourth-order Padé compact difference approximation for the spatial discretization; two HOC-ADI schemes with accuracy $O(\Delta t^2+h^4)$ are given. The two linearized ADI schemes apply extrapolation technique to the real coefficient of the nonlinear term to avoid iterating to solve. Unconditionally stable character is verified by linear Fourier analysis. The solution procedure consists of a number of tridiagonal matrix equations which make the computation cost effective. Numerical experiments are conducted to demonstrate the efficiency and accuracy, and linearized ADI schemes show less computational cost. All schemes given in this paper also can be used for two-dimensional linear Schrödinger equations.

MSC:

65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs

65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

35Q55 NLS equations (nonlinear Schrödinger equations)

Keywords:

cubic nonlinear Schrödinger equation; alternating direction implicit; high-order compact; extrapolation technique

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