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A symplectic proof of a theorem of Franks. (English) Zbl 1267.53093

Compos. Math. 148, No. 6, 1969-1984 (2012).

In two papers [Invent. Math. 108, No. 2, 403–418 (1992; Zbl 0766.53037); Trans. Am. Math. Soc. 348, No. 7, 2637–2662 (1996; Zbl 0862.58006)], *J. Franks* proved an important theorem in two-dimensional dynamics. His theorem says that every area-preserving homeomorphism of the two-sphere S^2 has either two or infinitely many periodic points. Later, Franks and Handel strengthened the theorem under the assumption of smoothness to add results on the growth rate of the periodic points.

The authors of this paper re-prove Franks' result in the smooth case using only tools from symplectic topology. The form their theorem takes is the following: Every Hamiltonian diffeomorphism ϕ of the symplectic manifold S^2 with sympletic form ω has either two or infinitely many periodic points. If ϕ has exactly two periodic points P and Q, then both are nondegenerate. In particular, both P and Q are elliptic fixed points of ϕ and their mean indices $\Delta(P)$ and $\Delta(Q)$ (in $\mathbb{R}/4\mathbb{Z}$) are irrational and satisfy $\Delta(P) + \Delta(Q) = 0 \bmod 4$.

The proof proceeds in several steps. First the authors prove that if a Hamiltonian diffeomorphism of S^2 has finitely many periodic points, then at least two of these points must have irrational mean indices. (This is a consequence of the theory of resonance relations for Hamiltonian diffeomorphisms.)

It then suffices to show that ϕ cannot have another periodic point with integer mean index. If such a point existed, then by blowing up a suitable iteration of ϕ and gluing the resulting map to itself, we achieve an area-preserving map of the torus. An argument using index relations and the Floer theory of symplectic diffeomorphisms show that such a map cannot exist.

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MSC:

53D40	Symplectic aspects of Floer homology and cohomology	Cited in **9** Documents
37J45	Periodic, homoclinic and heteroclinic orbits; variational methods, degree-theoretic methods (MSC2010)	
70H12	Periodic and almost periodic solutions for problems in Hamiltonian and Lagrangian mechanics	
37J05	Relations of dynamical systems with symplectic geometry and topology (MSC2010)	

Keywords:

periodic orbits; Floer homology; Hamiltonian flows; symplectic topology; diffeomorphisms of S^2

Full Text: DOI arXiv

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