

**Molev, A. I.**

**Feigin-Frenkel center in types  $B$ ,  $C$  and  $D$ .** (English) Zbl 1266.17016  
*Invent. Math.* 191, No. 1, 1-34 (2013).

The main result of the paper is an explicit construction of generators for the Feigin-Frenkel center of the affine vertex algebra at the critical level associated with a simple Lie algebra  $\mathfrak{g}$  of types  $B$ ,  $C$  and  $D$ . The construction is based on the properties of symmetrizers in the Brauer algebra which centralizes the action of the orthogonal or symplectic group on the tensor product of the vector representations in the context of the Schur-Weyl duality. The author also give formulae for central elements in terms of the Howe duality between involving symmetric and exterior powers of the natural representation of  $\mathfrak{g}$  and the dual action of  $\mathfrak{sl}_2$ .

As a corollary from the main result the author obtains explicit generators of the respective commutative subalgebras of the universal enveloping algebras  $U(\mathfrak{g}[t])$  and some other algebras. This leads to an explicit construction of higher order Hamiltonians of the Gaudin model associated with  $\mathfrak{g}$ . The author also constructs commutative Bethe-type subalgebras in the Yangians  $Y(\mathfrak{g})$  and shows that their graded images coincide with respective commutative subalgebras of  $U(\mathfrak{g}[t])$ .

Reviewer: [Volodymyr Mazorchuk \(Uppsala\)](#)

**MSC:**

**17B67** Kac-Moody (super)algebras; extended affine Lie algebras; toroidal Lie algebras Cited in 17 Documents

**Keywords:**

[Feigin-Frenkel center](#); [affine vertex algebra](#); [Schur-Weyl duality](#); [Brauer algebra](#); [trace](#); [Howe duality](#)

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