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Period integrals of CY and general type complete intersections. (English) Zbl 1276.32004
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The authors show a global Poincaré residue formula to study period integrals of families of complex manifolds. For any compact complex manifold X endowed with a linear system V^* of generically smooth CY hypersurfaces, the formula gives an expression for period integrals in terms of a canonical global meromorphic top form on X . For this construction the authors use the notion of a CY principal bundle and a classification of such rank one bundles. They generalize also the construction to CY and general type complete intersections. When V is an algebraic manifold having a sufficiently large automorphism group G and V^* is a linear representation of G , they construct a holonomic D -module that governs the periodic integrals. The construction is based in part on the theory of tautological systems developed in the paper [*B. H. Lian, R. Song and S.-T. Yau, J. Eur. Math. Soc. (JEMS) 15, No. 4, 1457–1483 (2013; Zbl 1272.14033)*]. The approach allows the authors to describe explicitly a Picard-Fuchs type system for complete intersection varieties of general type, as well as CY, in any Fano variety. They apply the results to toric manifolds and homogeneous spaces, as special examples and show that the period sheaves are governed by holonomic tautological systems. In the case of X a general homogeneous manifold, they obtain two different descriptions by using the Borel-Weil theorem and a theorem of Kostant and Lichtenstein, and they enumerate a set of generators for the tautological system. For X a toric manifold, the tautological systems turn out to be examples of GKZ hypergeometric systems and their extended versions. Moreover, the approach provides a new perspective of old examples such as CY complete intersections in a toric variety or partial flag variety.

Reviewer: [Anna Fino \(Torino\)](#)

MSC:

[32A27](#) Residues for several complex variables
[14J32](#) Calabi-Yau manifolds (algebraic-geometric aspects)
[32G20](#) Period matrices, variation of Hodge structure; degenerations

Cited in **2** Reviews
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Keywords:

[Calabi-Yau manifold](#); [linear system](#); [Poincaré residue formula](#); [period integral](#)

Full Text: [DOI](#) [arXiv](#)

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