

Zhang, Guang Yuan

The precise bound for the area-length ratio in Ahlfors' theory of covering surfaces. (English)

Zbl 1281.30021

Invent. Math. 191, No. 1, 197-253 (2013).

A key result in Ahlfors' theory of covering surfaces is that for any surface Σ over $\overline{\mathbb{C}} \setminus \{0, 1, \infty\}$, there exists a constant h such that the isoperimetric inequality

$$A(\Sigma) \leq hL(\partial\Sigma)$$

holds, where A, L denote area and length weighted according to multiplicity, respectively. This follows from an interpretation of the Second Fundamental Theorem of Nevanlinna Theory, and has further important consequences for complex analysis.

In the paper under review, the author obtains the smallest possible value of h , given by

$$h_0 = \max_{\theta \in [0, \pi/2]} \frac{(\pi + \theta)\sqrt{1 + \sin^2 \theta}}{\arctan\left(\frac{\sqrt{1 + \sin^2 \theta}}{\cos \theta}\right)}.$$

This is shown to be sharp by explicit construction of surfaces Σ_n with

$$\lim_{n \rightarrow \infty} \frac{A(\Sigma_n)}{L(\partial\Sigma_n)} = h_0.$$

In this well-written paper, the author proceeds by establishing the inequality for a certain family of surfaces \mathbb{F}^* , and then showing that this family well-approximates any other such surface in the sense of the quotient A/L .

Reviewer: [Alastair Fletcher \(Dekalb\)](#)

MSC:

- [30D20](#) Entire functions of one complex variable, general theory
- [30D35](#) Value distribution of meromorphic functions of one complex variable, Nevanlinna theory
- [30D45](#) Normal functions of one complex variable, normal families
- [51M25](#) Length, area and volume in real or complex geometry

Cited in **1** Review
Cited in **1** Document

Keywords:

[covering surface](#); [value distribution](#); [isoperimetric inequality](#); [spherical geometry](#)

Full Text: [DOI](#)

References:

- [1] Ahlfors, L.V.: Zur theorie der Überlagerung-Sflächen. Acta Math. 65, 157–194 (1935) · [Zbl 61.0365.03](#) · [doi:10.1007/BF02420945](#)
- [2] Alon, N., Pinchasi, A., Pinchasi, R.: An isoperimetric inequality in the universal cover of the punctured plane. Discrete Math. 308(23), 5691–5701 (2008) · [Zbl 1159.52009](#) · [doi:10.1016/j.disc.2007.10.033](#)
- [3] Bernstein, F.: Über die isoperimetrische eigenschaft des kreises auf der kugeloberfläche und in der ebene. Math. Ann. 60, 117–136 (1905) · [Zbl 36.0432.01](#) · [doi:10.1007/BF01447496](#)
- [4] Bonk, M., Eremenko, A.: Uniformly hyperbolic surfaces. Indiana Univ. Math. J. 49(1), 61–80 (2000) · [Zbl 0961.53021](#)
- [5] Burago, Y.D., Zalgaller, V.A.: Geometric inequalities. In: Grundlehren der Mathematischen Wissenschaften. Springer Series in Soviet Mathematics, vol. 285. Springer, Berlin (1988). Translated from the Russian by A.B. Sosinskiĭ · [Zbl 0633.53002](#)
- [6] Drasin, D.: The impact of Lars Ahlfors' work in value-distribution theory. Ann. Acad. Sci. Fenn., Ser. A I Math. 13(3), 329–353 (1988) · [Zbl 0673.30023](#) · [doi:10.5186/aasfm.1988.1325](#)

- [7] Dufresnoy, J.: Sur les domaines couverts par les valeurs d'une fonction méromorphe ou algébroïde. *Ann. Sci. Éc. Norm. Super.* (3) 58, 179–259 (1941) · [Zbl 67.0272.06](#)
- [8] Eremenko, A.: Ahlfors' contribution to the theory of meromorphic functions. In: *Lectures in Memory of Lars Ahlfors*, Haifa, 1996. *Israel Math. Conf. Proc.*, vol. 14, pp. 41–63. Bar-Ilan Univ., Ramat Gan (2000) · [Zbl 1020.30030](#)
- [9] Hayman, W.K.: *Meromorphic Functions*, Oxford (1964) · [Zbl 0115.06203](#)
- [10] Nevanlinna, R.: Zur theorie der meromorphen funktionen. *Acta Math.* 46, 1–99 (1925) · [Zbl 51.0254.05](#) · [doi:10.1007/BF02543858](#)
- [11] Nevanlinna, R.: *Analytic Functions*. *Die Grundlehren der Mathematischen Wissenschaften*, vol. 162. Springer, Berlin (1970). Translated from the second German edition by Phillip Emig · [Zbl 0199.12501](#)
- [12] Radó, T.: The isoperimetric inequality on the sphere. *Am. J. Math.* 57(4), 765–770 (1935) · [Zbl 61.0759.02](#) · [doi:10.2307/2371011](#)
- [13] Rickman, S.: *Quasiregular Mappings*. Springer, Berlin (1993). *Ergebnisse der Mathematik und Ihrer Grenzgebiete 3 Folge* · [Zbl 0816.30017](#)
- [14] Schnell, U., Segura Gomis, S.: Two problems concerning the area-perimeter ratio of lattice-point-free regions in the plane. *Beitrage Algebra Geom.* 37(1), 1–8 (1996) · [Zbl 0873.52017](#)
- [15] Stoilow, S.: *Leçons sur les Principes Topologiques de la Théorie des Fonctions Analytiques*. Gauthier-Villars, Paris (1956) · [Zbl 0072.07604](#)
- [16] Yang, L.: *Value Distribution Theory*. Springer, Berlin (1993) · [Zbl 0790.30018](#)
- [17] Zhang, G.Y.: Curves, Domains and Picard's theorem. *Bull. Lond. Math. Soc.* 34(2), 205–211 (2002) · [Zbl 1048.30024](#) · [doi:10.1112/S0024609301008712](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.