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Subharmonic functions in sub-Riemannian settings. (English) Zbl 1270.31002

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The authors give the mean value as well as the asymptotic characterization for $\mathcal{L}$-subharmonic functions, where $\mathcal{L}$ is a second order differential operator with non-negative characteristic form and well-behaved fundamental solution. An example of $\mathcal{L}$ can be the sub-Laplacian on Carnot groups. The authors also show how to approximate a subharmonic (in the sense of distributions) function by a smooth one.

Reviewer: Roman Urban (Wrocław)

MSC:

31C05 Harmonic, subharmonic, superharmonic functions on other spaces
35H20 Subelliptic equations
35J70 Degenerate elliptic equations

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