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Detachments of amalgamated 3-uniform hypergraphs: factorization consequences. (English)

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Summary: A detachment of a hypergraph $\mathcal{F}$ is a hypergraph obtained from $\mathcal{F}$ by splitting some or all of its vertices into more than one vertex. Amalgamating a hypergraph $\mathcal{G}$ can be thought of as taking $\mathcal{G}$, partitioning its vertices, then for each element of the partition squashing the vertices to form a single vertex in the amalgamated hypergraph $\mathcal{F}$.

In this paper, we use Nash-Williams lemma on laminar families to prove a detachment theorem for amalgamated 3-uniform hypergraphs, which yields a substantial generalization of previous amalgamation theorems by Hilton, Rodger, and Nash-Williams.

To demonstrate the power of our detachment theorem, we show that the complete 3-uniform n-partite multihypergraph $\lambda K^3_{m_1,\ldots,m_n}$ can be expressed as the union $\mathcal{G}_1 \cup \cdots \cup \mathcal{G}_k$ of k edge-disjoint factors, where for $i = 1, \ldots, k$, $\mathcal{G}_i$ is r_i-regular, if and only if:

(i) $m_i = m_j$ for all $1 \leq i, j \leq k$

(ii) $3 \mid r_i mn$ for each i, $1 \leq i \leq k$, and

(iii) $\sum_{i=1}^{k} r_i = \lambda\binom{n-1}{2}m^2$.

MSC:

05C65 Hypergraphs
05C70 Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)

Keywords:

amalgamations; detachments; 3-uniform hypergraphs; laminar families; factorization; decomposition

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