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2-starters, graceful labelings, and a doubling construction for the Oberwolfach problem.
(English) [Zbl 1258.05099](#)
J. Comb. Des. 20, No. 11-12, 483-503 (2012).

Summary: Every 1-rotational solution of a classic or twofold Oberwolfach problem (OP) of order n is generated by a suitable 2-factor (starter) of K_n or $2K_n$, respectively. It is shown that any starter of a twofold OP of order n gives rise to a starter of a classic OP of order $2n - 1$ (doubling construction). It is also shown that by suitably modifying the starter of a classic OP, one may obtain starters of some other OPs of the same order but having different parameters. The combination of these two constructions leads to lots of new infinite classes of solvable OPs. Still more classes can be obtained with the help of a third construction making use of the possible gracefulness of a graph whose connected components are cycles and at most one path. As one of the many applications, Hilton and Johnson's bound [*A. J. W. Hilton and M. Johnson*, *J. Lond. Math. Soc.*, II. Ser. 64, No. 3, 513–522 (2001; [Zbl 1012.05135](#))] $s \geq 5r - 1$ about the solvability of $OP(r, s)$ is improved to $s \geq \lfloor r/4 \rfloor + 10$ in the case of r even.

MSC:

- 05C70** Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.) Cited in 8 Documents
- 05C78** Graph labelling (graceful graphs, bandwidth, etc.)

Keywords:

[Oberwolfach problem](#); [1-rotational 2-factorization](#); [graceful labeling](#)

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