Pham, K.; **Marigo, J.-J.**

Stability of homogeneous states with gradient damage models: size effects and shape effects in the three-dimensional setting. (English) Zbl 1263.74046

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Summary: Considering a family of gradient-enhanced damage models and taking advantage of its variational formulation, we study the stability of homogeneous states in a full three-dimensional context. We show that gradient terms have a stabilizing effect, but also how those terms induce structural effects. We emphasize the great importance of the type of boundary conditions, the size and the shape of the body on the stability properties of such states.

MSC:

74R10 Brittle fracture
49J40 Variational inequalities
26A45 Functions of bounded variation, generalizations
47J30 Variational methods involving nonlinear operators

Keywords:

damage; gradient model; variational methods; stability; Rayleigh ratio

Full Text: DOI

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