Boscain, U.; Charlot, G.; Ghezzi, R.; Sigalotti, M.

Lipschitz classification of almost-Riemannian distances on compact oriented surfaces. (English) Zbl 1259.53031

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Summary: Two-dimensional almost-Riemannian structures are generalized Riemannian structures on surfaces for which a local orthonormal frame is given by a Lie bracket generating pair of vector fields that can become collinear. We consider the Carnot-Carathéodory distance canonically associated with an almost-Riemannian structure and study the problem of Lipschitz equivalence between two such distances on the same compact oriented surface. We analyze the generic case, allowing in particular for the presence of tangency points, i.e., points where two generators of the distribution and their Lie bracket are linearly dependent. The main result of the paper provides a characterization of the Lipschitz equivalence class of an almost-Riemannian distance in terms of a labeled graph associated with it.

MSC:

53C17 Sub-Riemannian geometry

53C15 General geometric structures on manifolds (almost complex, almost product structures, etc.)

54E35 Metric spaces, metrizability

Keywords:

almost-Riemannian surface; Lipschitz equivalence; Carnot-Carathéodory distance; sub-Riemannian geometry

Full Text: DOI

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