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Algebraic hulls of solvable groups and exponential iterated integrals on solvmanifolds. (English) Zbl 1280.22013

Geom. Dedicata 162, 263-270 (2013).

Let G be a simply connected Lie group with Lie algebra $\mathfrak{g}$, and Γ a cocompact discrete subgroup of G. If G is nilpotent, then Chen's (closed) iterated integrals induced from $\bigwedge \mathfrak{g}_{\mathbb{C}}^*$ represent the coordinate ring of the Malcev completion of Γ. In this paper, a generalized case is discussed where G is a solvable Lie group and (hence) Γ is a torsion-free polycyclic group. The author applies the theory of *C. Miller* [Topology 44, No. 2, 351–373 (2005; Zbl 1149.57315)] and shows that the coordinate ring of the algebraic hull of Γ is represented by Miller's (closed) exponential iterated integrals induced from a $\mathbb{Z}$-lattice of $\mathfrak{g}_{\mathbb{C}}^*$ and $\bigwedge \mathfrak{g}_{\mathbb{C}}^*$.

Reviewer: Hiroaki Nakamura (Osaka)

MSC:

22E25 Nilpotent and solvable Lie groups
14H30 Coverings of curves, fundamental group
55P62 Rational homotopy theory
20F99 Special aspects of infinite or finite groups

Keywords:

exponential iterated integral; algebraic hull; solvmanifold

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References:

[1] Chen K.T.: Iterated path integrals. Bull. Am. Math. Soc. 83(5), 831–879 (1977) · Zbl 0389.58001 · doi:10.1090/S0002-9904-1977-14320-6

[2] Chen K.T.: Reduced bar constructions on de Rham complexes. In: Heller, A., Tierney, A. (eds.) Algebra Topology, and Category Theory, pp. 19–32. Academic Press, London (1977)

[3] Hasegawa K.: Minimal models of nilmanifolds. Proc. Am. Math. Soc. 106(1), 65–71 (1989) · Zbl 0691.53040 · doi:10.1090/S0002-9939-1989-0946638-X

[4] Hattori A.: Spectral sequence in the de Rham cohomology of fibre bundles. J. Fac. Sci. Univ. Tokyo Sect. I 8, 289–331 (1960) · Zbl 0099.18003

[5] Hochschild G., Mostow G.D.: Pro-affine algebraic groups. Am. J. Math. 91, 1127–1140 (1969) · Zbl 0213.22702 · doi:10.2307/2373319

[6] Miller C.: Exponential iterated integrals and the relative solvable completion of the fundamental group of a manifold. Topology 44, 351–373 (2005) · Zbl 1149.57315 · doi:10.1016/j.top.2004.10.005

[7] Mostow G.D.: Factor spaces of solvable groups. Ann. Math. 60(2), 1–27 (1954) · Zbl 0057.26103 · doi:10.2307/1969700

[8] Mostow G.D.: Fully reducible subgroups of algebraic groups. Am. J. Math. 78, 200–221 (1956) · Zbl 0073.01603 · doi:10.2307/2372490

[9] Mostow G.D.: Representative functions on discrete groups and solvable arithmetic subgroups. Am. J. Math. 92, 1–32 (1970) · Zbl 0205.04406 · doi:10.2307/2373495

[10] Nomizu K.: On the cohomology of compact homogeneous spaces of nilpotent Lie groups. Ann. Math. 59(2), 531–538 (1954) · Zbl 0058.02202 · doi:10.2307/1969716

[11] Raghunathan M.S: Discrete Subgroups of Lie Groups. Springer, New York (1972) · Zbl 0254.22005

[12] Reed B.E.: Representations of solvable Lie algebras. Mich. Math. J. 16, 227–233 (1969) · Zbl 0204.36002 · doi:10.1307/mmj/1029000266

[13] Vinberg E.B., Gorbatsevich V.V., Schvartsman O.V.: Discrete subgroups of Lie groups, Lie groups and Lie algebras II. In: Onishchik, A.L., Vinberg, E.B. (eds.) Encyclopaedia Mathematical Sciences, vol. 21, pp. 1–123. Springer-Verlag, New York-Berlin (2000)