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Some general results about proof normalization. (English) Zbl 1255.03050

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Summary: In this paper, we provide a general setting under which results of normalization of proof trees such as, for instance, the locality result in equational reasoning and the cut-elimination property in sequent or natural deduction calculi, can be unified and generalized. This is achieved by giving simple conditions which are sufficient to ensure that such normalization results hold, and which can be automatically checked since they are syntactical. These conditions are based on basic properties of elementary combinations of inference rules which ensure that the induced global proof tree transformation processes do terminate.

MSC:

[03F05](#) Cut-elimination and normal-form theorems
[03B22](#) Abstract deductive systems
[03F07](#) Structure of proofs

Keywords:

formal systems; proof tree transformation; weak and strong normalization; cut elimination; rewriting

Software:

CoCasl; Coq; Isabelle

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