Gerasimov, Victor

Floyd maps for relatively hyperbolic groups. (English) Zbl 1276.20050

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Given a finitely generated group G and a finite generating set, the edges in the Cayley graph have length 1. W. J. Floyd conformally deformed the metric to get a metric space with finite diameter. Let $f\colon \mathbb{N} \to (0,\infty)$ be a non-increasing function with $\sum_{n=0}^{\infty} f(n) < \infty$ such that, for any $k \geq 1$, there are $M, N > 0$ so that $Mf(r) \leq f(kr) \leq Nf(r)$ for all r. Define a new length metric on the Cayley graph by giving an edge e length $f(n)$ if the distance from e to the identity element is n. This new metric space Γ_f has finite diameter. Its completion $\overline{\Gamma}_f$ is a Floyd completion and $\partial_f G = \overline{\Gamma}_f \setminus \Gamma_f$ is a Floyd boundary. The left translation action of G on itself induces actions on $\overline{\Gamma}_f$ and $\partial_f G$ by biLipschitz maps.

Floyd proved that if G is a geometrically finite Kleinian group and $f(n) = \frac{1}{n^2+1}$, then there is a continuous equivariant map from the Floyd boundary to the limit set. The main result of the current paper generalizes this result to the case of relatively hyperbolic groups. Given a relatively hyperbolic group G, there is some $0 < \lambda < 1$, such that for the function $f(n) = \lambda^n$, there is a continuous equivariant map from the Floyd boundary $\partial_f G$ to the Bowditch boundary. This in particular implies the Floyd boundary is non-trivial.

To facilitate the proof of the main theorem, the author proposes two new definitions of relatively hyperbolic groups, which are equivalent to the known ones. They serve as bridges between the geometric definition (actions on fine graphs) and the dynamics definition (convergence action).

Another interesting result is the so called Attractor Sum Theorem. It says the following: given a convergence action of a locally compact group G on a compactum Λ, and a proper cocompact action of G on a locally compact Hausdorff space Ω, there is a unique topology on the disjoint union $T := \Lambda \cup \Omega$ extending the topologies on Λ and Ω, such that T is compact Hausdorff and the action of G on T is also a convergence action.

Reviewer: Xiangdong Xie (Bowling Green)

MSC:

20F65	Geometric group theory
20F67	Hyperbolic groups and nonpositively curved groups
22D05	General properties and structure of locally compact groups
57M07	Topological methods in group theory
20F05	Generators, relations, and presentations of groups
30F40	Kleinian groups (aspects of compact Riemann surfaces and uniformization)

Keywords:

Floyd completions; relatively hyperbolic groups; convergence group actions; locally compact groups; Cayley graphs; geometrically finite Kleinian groups

Full Text: DOI

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