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On starshaped rearrangement and applications. (English) Zbl 0612.35033
Trans. Am. Math. Soc. 296, 377-386 (1986).

Author's summary: A radial symmetrization technique is investigated and new properties are proven. The method transforms functions u into new functions u^* with starshaped level sets and is therefore called a starshaped rearrangement. This rearrangement is in general not equimeasurable, a circumstance with some surprising consequences. The method is then applied to certain variational and free boundary problems and yields new results on the geometrical properties of solutions of these problems. In particular, the Lipschitz continuity of free boundaries can now be easily obtained in a new fashion.

Reviewer: A.D.Osborne

MSC:

- [35J20](#) Variational methods for second-order elliptic equations
- [35R35](#) Free boundary problems for PDEs
- [35B05](#) Oscillation, zeros of solutions, mean value theorems, etc. in context of PDEs
- [26D20](#) Other analytical inequalities

Cited in **3** Documents

Keywords:

radial symmetrization; starshaped level sets; rearrangement; free boundary; geometrical properties; Lipschitz continuity

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