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s-vertex pancyclic index. (English) Zbl 1256.05125

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Summary: A graph G is vertex pancyclic if for each vertex $v \in V(G)$, and for each integer k with $3 \leq k \leq |V(G)|$, G has a k-cycle C_k such that $v \in V(C_k)$. Let $s \geq 0$ be an integer. If the removal of at most s vertices in G results in a vertex pancyclic graph, we say G is an s-vertex pancyclic graph. Let G be a simple connected graph that is not a path, cycle or $K_{1,3}$. Let $l(G) = \max\{m : G$ has a divalent path of length m that is not both of length 2 and in a $K_3\}$, where a divalent path in G is a path whose interval vertices have degree two in G. The s-vertex pancyclic index of G, written $vp_s(G)$, is the least nonnegative integer m such that $L^m(G)$ is s-vertex pancyclic. We show that for a given integer $s \geq 0$,

$$vp_s(G) \leq \begin{cases} l(G) + s + 1: & \text{if } 0 \leq s \leq 4 \\ l(G) + \lceil \log_2(s-2) \rceil + 4: & \text{if } s \geq 5 \end{cases}.$$

And we improve the bound for essentially 3-edge-connected graphs. The lower bound and whether the upper bound is sharp are also discussed.

MSC:

05C38 Paths and cycles

05C76 Graph operations (line graphs, products, etc.)

Keywords:

vertex pancyclic graph; vertex pancyclic index; triangular graph; line graph

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