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Regularization of local CR-automorphisms of real-analytic CR-manifolds. (English)

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The authors study the class of generic real-analytic CR-submanifolds of a complex vector space E with the following property: For every point a of the manifold, the Lie algebra of germs of infinitesimal real-analytic CR automorphisms at a is finite-dimensional and its complexification contains all constant vector fields and the Euler vector field. This class includes the Levi non-degenerate CR-quadratics and a number of Levi degenerate tube manifolds.

The main results are: (I) The germs of infinitesimal real-analytic CR-automorphisms extend to global polynomial vector fields. (II) The local real-analytic CR-automorphisms extend to birational transformations of E . In particular, the authors specify conditions under which the local birational transformations of the CR-manifold form a group. (III) The group generated by those birational transformations can be realised as a group of projective transformations.

These results can be viewed as a generalization and plausible explanation of the heuristic formula for quadric automorphisms described by *V. V. Ezhov* and *G. Schmalz* in [J. Geom. Anal. 11, No. 3, 441–467 (2001; Zbl 1042.32014)].

Reviewer: [Gerd Schmalz \(Armidale\)](#)

MSC:

[32V40](#) Real submanifolds in complex manifolds
[14M17](#) Homogeneous spaces and generalizations

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