Bonfert-Taylor, Petra; Matsuzaki, Katsuhiko

Large and small covers of a hyperbolic manifold. (English) Zbl 1272.30062

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For a discrete subgroup Γ of the isometry group of the hyperbolic space $\mathbb{H}^{n+1}$, we denote by $\delta(\Gamma)$ the exponent of convergence of its Poincaré series. By work of *C. J. Bishop* and *P. W. Jones* [Acta Math. 179, No. 1, 1–39 (1997; Zbl 0921.30032)] it is known that $\delta(\Gamma)$ coincides with the Hausdorff dimension of the conical limit set of Γ.

In the paper under review, the authors focus on the behaviour of δ under taking non-trivial normal subgroups. *K. Falk* and *B. O. Stratmann* [Tohoku Math. J., II. Ser. 56, No. 4, 571–582 (2004; Zbl 1069.30070)] showed that if $\hat{\Gamma}$ is a non-trivial normal subgroup of a non-elementary Γ, then $\delta(\hat{\Gamma}) \geq \delta(\Gamma)/2$. In the present paper, it is shown that in the case when $n = 1$ and Γ is a Fuchsian group corresponding to a closed hyperbolic surface, this inequality is in a sense best possible: there is a sequence of normal subgroups Γ_i with $\delta(\Gamma_i)$ tending to $1/2$. Furthermore, it is also shown that when Γ is non-elementary and convex cocompact for arbitrary n, for any non-trivial normal subgroup $\hat{\Gamma}$, the strict inequality $\delta(\hat{\Gamma}) > \delta(\Gamma)/2$ holds.

In contrast to these results, the authors also show that in the 3-dimensional cocompact case, there is a normal subgroup with large δ: when $\mathbb{H}^3/\Gamma$ is a closed hyperbolic 3-manifold fibring over a circle, for any $\epsilon > 0$ there is a Schottky subgroup G of Γ with $\delta(G) > 2 - \epsilon$.

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MSC:

30F40 Kleinian groups (aspects of compact Riemann surfaces and uniformization)

57M50 General geometric structures on low-dimensional manifolds

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Kleinian group; hyperbolic space; exponent of convergence

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