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On the large time behavior of solutions of Hamilton-Jacobi equations associated with nonlinear boundary conditions. (English) Zbl 1282.70037

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Summary: We study the large time behavior of solutions of first-order Hamilton-Jacobi equations set in a bounded domain with nonlinear von Neumann boundary conditions, including the case of dynamical boundary conditions. We establish general convergence results for viscosity solutions of these Cauchy-von Neumann problems by using two fairly different methods: the first one relies only on partial differential equations methods, which provides results even when the Hamiltonians are not convex, and the second one is an optimal control/dynamical system approach, named the "weak KAM approach", which requires the convexity of Hamiltonians and gives formulas for asymptotic solutions based on Aubry-Mather sets.

MSC:

70H20 Hamilton-Jacobi equations in mechanics
35Q70 PDEs in connection with mechanics of particles and systems of particles
49L25 Viscosity solutions to Hamilton-Jacobi equations in optimal control and differential games

Keywords:

Cauchy-von Neumann problem; convergence; weak KAM approach; Aubry-Mather set

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