Moser, Jürgen

Minimal solutions of variational problems on a torus. (English) Zbl 0609.49029

Ann. Inst. Henri Poincaré, Anal. Non Linéaire 3, 229-272 (1986).

The author considers a special class of extremals (minimal solutions) of variational problems on the torus T^{n+1}, defined as the quotient of its universal covering manifold R^{n+1} by the group Z^{n+1}. Consider the n-dimensional hypersurfaces in R^{n+1} represented as the graph of a function u on R^n by (1) $x_{n+1} = u(x)$, $x \in R^n$. The function $u \in W^{1,2}_{loc}(R^n)$ is a minimal solution for the variational problem $\int_{\mathbb{R}^n} f(x, u, u_x)dx$, if $\int_{\mathbb{R}^n}[f(x, u+g, u_x g_x) - f(x, u, u_x)]dx \geq 0$, $g \in W^{1,2}_{comp}(R^n)$. He introduces the notion of no selfintersections: the surface (1) has no selfintersection on T^{n+1} if $[u(x + j) - j_{n+1}] - u(x) = \tau_{\bar{j}} u(x)$, $(j, j_{n+1}) = \bar{j} \in Z^{n+1}$, has constant sign, i.e. is for all x either positive or negative or identically zero. The author studies the properties of the set of minimal solutions without selfintersections, he proves a priori estimates for them and establishes their existence. Moreover, the relationships with folitations of extremals are pointed out. The tools of the proofs are essentially from calculus of variations. Finally, some open problems are given.

Reviewer: E.Mascolo

MSC:

49Q20 Variational problems in a geometric measure-theoretic setting
53A07 Higher-dimensional and -codimensional surfaces in Euclidean and related n-spaces

Keywords:

variational problems on the torus; minimal solutions without selfintersections; folitations of extremals

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