

**Doha, E. H.; Bhrawy, A. H.; Hafez, R. M.**

**On shifted Jacobi spectral method for high-order multi-point boundary value problems.**  
(English) [Zbl 1251.65112](#)  
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**Summary:** This paper reports a spectral tau method for numerically solving multi-point boundary value problems (BVPs) of linear high-order ordinary differential equations. The construction of the shifted Jacobi tau approximation is based on conventional differentiation. This use of differentiation allows the imposition of the governing equation at the whole set of grid points and the straight forward implementation of multiple boundary conditions. Extension of the tau method for high-order multi-point BVPs with variable coefficients is treated using the shifted Jacobi Gauss-Lobatto quadrature. The shifted Jacobi collocation method is developed for solving nonlinear high-order multi-point BVPs. The performance of the proposed methods is investigated by considering several examples. Accurate results and high convergence rates are achieved.

**MSC:**

- [65L10](#) Numerical solution of boundary value problems involving ordinary differential equations
- [34B05](#) Linear boundary value problems for ordinary differential equations
- [34B10](#) Nonlocal and multipoint boundary value problems for ordinary differential equations
- [34B15](#) Nonlinear boundary value problems for ordinary differential equations
- [65L60](#) Finite element, Rayleigh-Ritz, Galerkin and collocation methods for ordinary differential equations
- [65L20](#) Stability and convergence of numerical methods for ordinary differential equations

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**Keywords:**

Jacobi spectral method; multi-point boundary value problem; high-order differential equation; nonlinear boundary value problems; tau method; collocation method; shifted Jacobi polynomials; numerical examples; nonlinear problems; linear problems; Gauss-Lobatto quadrature; convergence

**Full Text:** [DOI](#)

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