Zhou, Shiwei; **Wang, Michael Yu**

Multimaterial structural topology optimization with a generalized Cahn-Hilliard model of multiphase transition. (English) Zbl 1245.74077

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Summary: We describe a phase field method for the optimization of multimaterial structural topology with a generalized Cahn-Hilliard model. Similar to the well-known simple isotropic material with penalization method, the mass concentration of each material phase is considered a design variable. However, a variational approach is taken with the Cahn-Hilliard theory to define a thermodynamic model, taking into account the bulk energy and interface energy of the phases and the elastic strain energy of the structure. As a result, the structural optimization problem is transformed into a phase transition problem defined by a set of nonlinear parabolic partial differential equations. The generalized Cahn-Hilliard model regularizes the original ill-posed topology optimization problem and provides the flexibility of topology changes with interface coalescence and break-up due to phase separation and coarsening. We employ a powerful multigrid algorithm and extend it to include four material phases for numerical solution of the Cahn-Hilliard equations. We demonstrate our approach through several 2-D and 3-D examples to minimize mean compliance of the multimaterial structures.

MSC:

74P15 Topological methods for optimization problems in solid mechanics
74H15 Numerical approximation of solutions of dynamical problems in solid mechanics
74G65 Energy minimization in equilibrium problems in solid mechanics

Keywords:

Cahn-Hilliard model; multiphase transition

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