Young, D. L.; Lin, Y. C.; Fan, C. M.; Chiu, C. L.

The method of fundamental solutions for solving incompressible Navier-Stokes problems. (English) Zbl 1244.76104

Eng. Anal. Bound. Elem. 33, No. 8-9, 1031-1044 (2009).

Summary: A novel meshless numerical procedure based on the method of fundamental solutions (MFS) is proposed to solve the primitive variables formulation of the Navier-Stokes equations. The MFS is a meshless method since it is free from the mesh generation and numerical integration. We will transform the Navier-Stokes equations into simple advection-diffusion and Poisson differential operators via the operator-splitting scheme or the so-called projection method, instead of directly using the more complicated fundamental solutions (Stokeslets) of the unsteady Stokes equations. The resultant velocity advection-diffusion equations and the pressure Poisson equation are then calculated by using the MFS together with the Eulerian-Lagrangian method (ELM) and the method of particular solutions (MPS). The proposed meshless numerical scheme is a first attempt to apply the MFS for solving the Navier-Stokes equations in the moderate-Reynolds-number flow regimes. The lid-driven cavity flows at the Reynolds numbers up to 3200 for two-dimensional (2D) and 1000 for three-dimensional (3D) are chosen to validate the present algorithm. Through further simulating the flows in the 2D circular cavity with an eccentric rotating cylinder and in the 3D cube with a fixed sphere inside, we are able to demonstrate the advantages and flexibility of the proposed meshless method in the irregular geometry and multi-dimensional flows, even though very coarse node points are used in this study as compared with other mesh-dependent numerical schemes.

MSC:

76M28 Particle methods and lattice-gas methods
76D05 Navier-Stokes equations for incompressible viscous fluids
65M80 Fundamental solutions, Green's function methods, etc. for initial value and initial-boundary value problems involving PDEs

Keywords:

Navier-Stokes equations; meshless numerical method; method of fundamental solutions; method of particular solutions; Eulerian-Lagrangian method; operator-splitting method

Full Text: DOI

References:

[1] Fairweather, G.; Karageorghis, A., The method of fundamental solutions for elliptic value problems, Adv comput math, 9, 69-95, (1998) · Zbl 0922.65074

[2] Golberg, M.A.; Chen, C.S., The method of fundamental solutions for potential, Helmholtz and diffusion problems, (), 103-176 · Zbl 0945.65130

[3] Kupradze, V.D.; Aleksidze, M.A., The method of fundamental equations for the approximate solution of certain boundary value problem, Zh vychisl mat, 4, 4, 82-126, (1964) · Zbl 0154.17604

[4] Mathon, R.; Johnston, R.L., The approximate solution of elliptic boundary-value problems by fundamental solutions, SIAM J numer anal, 14, 638-650, (1977) · Zbl 0368.65058

[5] Bogomolny, A., Fundamental solutions method for elliptic boundary value problems, SIAM J numer anal, 22, 644-669, (1985) · Zbl 0579.65121

[6] Karageorghis, A., The method of fundamental solutions for the calculation of the eigenvalues of the Helmholtz equation, Appl math let, 14, 837-842, (2001) · Zbl 0984.65111

[7] Karageorghis, A.; Fairweather, G., The method of fundamental solutions for the numerical solution of the biharmonic equation, J comput phys, 69, 434-459, (1987) · Zbl 0618.65108

[8] Young, D.L.; Jane, S.J.; Fan, C.M.; Murugesan, K.; Tsai, C.C., The method of fundamental solutions for 2D and 3D Stokes problems, J comput phys, 211, 1-8, (2006) · Zbl 1160.76332

[9] Golberg, M.A., The method of fundamental solutions for Poisson's equations, Eng anal bound elem, 16, 205-213, (1995)

[10] Golberg, M.A.; Muleshkov, A.S.; Chen, C.S.; Cheng, A.H.-D., Polynomial particular solutions for certain partial differential operators, Numer methods partial differential equation, 19, 1, 112-133, (2002) · Zbl 1019.65096

[11] Young, D.L.; Tsai, C.C.; Murugesan, K.; Fan, C.M.; Chen, C.W., Time-dependent fundamental solutions for homogeneous diffusion problems, Eng anal boundary elem, 29, 1463-1473, (2004) · Zbl 1098.76622

[12] Young, D.L.; Fan, C.M.; Tsai, C.C.; Chen, C.W.; Murugesan, K., Solution of advection – diffusion equation using the eulerian – lagrangian method of fundamental solutions, Int math forum, 1, 14, 687-706, (2006) · Zbl 1143.65384

[13] Young, D.L.; Fan, C.M.; Hu, S.P.; Atluri, S.N., The eulerian – lagrangian method of fundamental solutions for two-dimensional unsteady burgers' equations, Eng anal boundary elem, 32, 395-412, (2008) · Zbl 1244.76096

[14] Tsai, C.C.; Young, D.L.; Fan, C.M.; Chen, C.W., MFS with time-dependent fundamental solutions for unsteady Stokes equations, Eng anal boundary elem, 30, 897-908, (2006) · Zbl 1195.76324

[15] Young, D.L.; Chen, C.W.; Fan, C.M., The method of fundamental solutions for low Reynolds number flows with moving rigid body, (), 181-206

[16] Chorin, A.J., Numerical solution of the navier – stokes equations, Math comput, 22, 745-762, (1968) · Zbl 0198.50103

[17] Temam, R., Une méthode d'approximation de la solution des équations de navier – stokes, Bull soc math France, 98, 115-152, (1968) · Zbl 0181.18903

[18] Patankar, S.V.; Spalding, D.B., A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows, Int J heat mass transfer, 15, 1787-1806, (1972) · Zbl 0246.76080

[19] Goldberg, D.; Ruas, V., A numerical study of projection algorithms in the finite element simulation of three-dimensional viscous incompressible flow, Int J numer methods fluids, 30, 233-256, (1999) · Zbl 0945.76045

[20] Brown, D.; Cortez, R.; Minion, M., Accurate projection method for incompressible navier – stokes equations, J comput phys, 168, 464-499, (2001) · Zbl 1153.76339

[21] Chang, W.; Giraldo, F.; Perot, B., Analysis of an exact fractional step method, J comput phys, 180, 183-199, (2002) · Zbl 1130.76394

[22] Duchon, J., Splines minimizing rotation invariant seminorms in Sobolev spaces, (), 85-110

[23] Golberg, M.A.; Chen, C.S., Discrete projection methods for integral equations, (1997), Computational Mechanics Publication Southampton

[24] Burggraf, O.R., Analytical and numerical studies of the structure of steady separated flow, J fluid mech, 24, 113-151, (1966)

[25] Ghia, U.; Ghia, K.N.; Shin, C.T., High-\textit{re} solutions for incompressible flow using the navier – stokes equations and a multigrid method, J comput phys, 48, 387-411, (1982) · Zbl 0511.76031

[26] Jiang, B.N.; Lin, T.L.; Provinelli, L.A., Large-scale computation of incompressible viscous flow by least-square finite element method, Compu methods appl mech eng, 114, 213-231, (1994)

[27] Tsai, C.C.; Lin, Y.C.; Young, D.L.; Atluri, S.N., Investigations on the accuracy and condition number for the method of fundamental solutions, Cmes, 16, 2, 103-114, (2006)

[28] Young, D.L.; Chiu, C.L.; Fan, C.M., A hybrid Cartesian/immersed-boundary finite-element method for simulating heat and flow patterns in a two-roll mill, Numer heat transfer, B, 51, 251-274, (2007)

[29] Chiu, C.L.; Fan, C.M.; Young, D.L., 3D hybrid Cartesian/immersed-boundary finite-element analysis of heat and flow patterns in a two-roll mill, Int J heat mass transfer, 52, 7-8, 1677-1689, (2009) · Zbl 1157.80314