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Hyponormality and subnormality of block Toeplitz operators. (English) Zbl 1248.47029
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An operator T on a Hilbert space is said to be normal if $T^*T - TT^* = 0$, hyponormal if $T^*T - TT^* \geq 0$, and subnormal if it has a normal extension. The paper is devoted to hyponormality and subnormality of block Toeplitz operators acting on the vector-valued Hardy space H^2 of the unit circle. A function $\varphi \in L^\infty$ is said to be of bounded type (or in the Nevanlinna class) if there are analytic functions $\psi_1, \psi_2 \in H^\infty$ such that $\varphi = \psi_1/\psi_2$ almost everywhere on the unit circle. The first main result of the paper is a criterion for the hyponormality of block Toeplitz operators with bounded type symbols. The second main result is related to the Halmos problem: is every subnormal Toeplitz operator either normal or analytic? It is shown that, if Φ is a matrix-valued rational function whose co-analytic part has a coprime factorization, then every hyponormal Toeplitz operator T_Φ whose square is also hyponormal must be either normal or analytic. Third, using the subnormality theory of block Toeplitz operators, the authors give an answer to a Toeplitz subnormal completion problem. Finally, some open problems are listed.

Reviewer: [Alexei Yu. Karlovich \(Lisboa\)](#)

MSC:

- [47B35](#) Toeplitz operators, Hankel operators, Wiener-Hopf operators
- [47B20](#) Subnormal operators, hyponormal operators, etc.
- [47A13](#) Several-variable operator theory (spectral, Fredholm, etc.)
- [30H10](#) Hardy spaces
- [47A20](#) Dilations, extensions, compressions of linear operators
- [47A57](#) Linear operator methods in interpolation, moment and extension problems

Cited in **16** Documents

Keywords:

block Toeplitz operator; hyponormal operator; square-hyponormal operator; subnormal operator; bounded type function; rational function; trigonometric polynomial; Toeplitz subnormal completion problem

Full Text: [DOI](#) [arXiv](#)

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