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**An active strain electromechanical model for cardiac tissue.** (English) Zbl 1242.92016  
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**Summary:** We propose a finite element approximation of a system of partial differential equations describing the coupling between the propagation of the electrical potential and large deformations of the cardiac tissue. The underlying mathematical model is based on the active strain assumption, in which it is assumed that there is a multiplicative decomposition of the deformation tensor into a passive and active part, the latter carrying the information of the electrical potential propagation and anisotropy of the cardiac tissue into the equations of either incompressible or compressible nonlinear elasticity, governing the mechanical response of the biological material. In addition, by changing from a Eulerian to a Lagrangian configuration, the bidomain or monodomain equations modeling the evolution of the electrical propagation exhibit a nonlinear diffusion term. Piecewise quadratic finite elements are employed to approximate the displacements field, whereas for pressure, electrical potentials and ionic variables are approximated by piecewise linear elements. Various numerical tests performed with a parallel finite element code illustrate that the proposed model can capture some important features of the electromechanical coupling and show that our numerical scheme is efficient and accurate.

**MSC:**

92C30 Physiology (general)  
92C05 Biophysics  
78A70 Biological applications of optics and electromagnetic theory  
92C10 Biomechanics  
35K57 Reaction-diffusion equations  
65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs  
35Q92 PDEs in connection with biology, chemistry and other natural sciences

Cited in **26** Documents

**Keywords:**

cardiac electromechanical coupling; bidomain equations; reaction-diffusion problem; nonlinear elasticity; finite elements

**Software:**

FreeFem++; LifeV

**Full Text:** [DOI](#)

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