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Periodic solutions of nonlinear delay differential equations using spectral element method.

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Summary: We extend the temporal spectral element method further to study the periodic orbits of general autonomous nonlinear delay differential equations (DDEs) with one constant delay. Although we describe the approach for one delay to keep the presentation clear, the extension to multiple delays is straightforward. We also show the underlying similarities between this method and the method of collocation. The spectral element method that we present here can be used to find both the periodic orbit and its stability. This is demonstrated with a variety of different examples, namely, the delayed versions of Mackey-Glass equation, Van der Pol equation, and Duffing equation. For each example, we show the method's convergence behavior using both p and h refinement and we provide comparisons between equal size meshes that have different distributions.

MSC:

65L03	Numerical methods for functional-differential equations
65L60	Finite element, Rayleigh-Ritz, Galerkin and collocation methods for ordinary differential equations
65L20	Stability and convergence of numerical methods for ordinary differential equations
65L12	Finite difference and finite volume methods for ordinary differential equations
65L50	Mesh generation, refinement, and adaptive methods for ordinary differential equations
34K28	Numerical approximation of solutions of functional-differential equations (MSC2010)

Keywords:

mesh refinement; numerical examples; spectral element method; periodic orbits; autonomous nonlinear delay differential equations; stability; Mackey-Glass equation; Van der Pol equation; Duffing equation; convergence

Software:

AUTO; RADAR5

Full Text: DOI

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