Coulbois, Thierry; Hilion, Arnaud
Botany of irreducible automorphisms of free groups. (English) Zbl 1259.20031
Pac. J. Math. 256, No. 2, 291-307 (2012).

Let F_N be a free group of rank N. An outer automorphism Φ of F_N is fully irreducible (iwip) if no positive power Φ^n fixes a proper free factor of F_N. In this paper the authors classify the fully irreducible outer automorphisms of a free group.

Before quoting their main result some definitions and terminology are needed.

The group of outer automorphisms $\mathrm{Out}(F_N)$ acts on the outer space CV_N and its boundary ∂CV_N [see *K. Vogtmann*, Geom. Dedicata 94, 1-31 (2002; Zbl 1017.20035)]. An iwip outer automorphism Φ has a unique attracting fixed tree $[T_\Phi]$ and a unique repelling fixed tree $[T_{\Phi^{-1}}]$ in the boundary of outer space.

The free group F_N may be realized as the fundamental group of a surface S with boundary. If Φ comes from a pseudo-Anosov mapping class on S, then its limit trees T_Φ and $T_{\Phi^{-1}}$ are called surface trees and such an iwip outer automorphism Φ is called geometric. If Φ does not come from a pseudo-Anosov mapping class and if T_Φ is geometric then Φ is called parageometric.

For a tree T in the boundary of outer space with dense orbits, the limit set $\Omega \subseteq \overline{T}$ ($\overline{T}$ is the metric completion of T) consists of points of $\overline{T}$ with at least two pre-images by the map $\mathfrak{D}\colon \partial F_N \to \overline{T} \cup \partial T$ introduced by *G. Levitt* and *M. Lustig* [J. Inst. Math. Jussieu 2, No. 1, 59-72 (2003; Zbl 1034.20038)]. If $T \subseteq \Omega$, the tree T is called of surface type. If Ω is totally disconnected, the tree T is called of Levitt type.

For a tree T in ∂CV_N with dense orbits in [*T. Coulbois* and *A. Hilion*, "Rips induction: index of the dual lamination of an $\mathbb{R}$-tree", arXiv:1002.0972] are summarized the above properties and is given the definition.

The tree T is:

— a surface tree if it is both geometric and of surface type;

— Levitt if it is geometric and of Levitt type;

— pseudo-surface if it is not geometric and of surface type;

— pseudo-Levitt if it is not geometric and of Levitt type.

The following theorem is the main result of this paper.

Theorem: Let Φ be an iwip outer automorphism of F_N. Let T_Φ and $T_{\Phi^{-1}}$ be its attracting and repelling trees. Then exactly one of the following occurs:

1. The trees T_Φ and $T_{\Phi^{-1}}$ are surface trees. Equivalently, Φ is geometric.
2. The tree T_Φ is Levitt and the tree $T_{\Phi^{-1}}$ is pseudo-surface. Equivalently, Φ is parageometric.
3. The tree $T_{\Phi^{-1}}$ is Levitt, and the tree T_Φ is pseudo-surface. Equivalently, Φ^{-1} is parageometric.
4. The trees T_Φ and $T_{\Phi^{-1}}$ are pseudo-Levitt.

Another interesting result in the paper, based on the property that iwip automorphisms can be represented by (absolute) train-track maps, is the

Theorem: Let $\Phi \in \mathrm{Out}(F_N)$ be an iwip outer automorphism. The attracting tree T_Φ is indecomposable.

For the indecomposability of a tree see *V. Guirardel* [Ann. Inst. Fourier 58, No. 1, 159-211 (2008; Zbl 1187.20020)].

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MSC:

20E05 Free nonabelian groups
20E36 Automorphisms of infinite groups
20F65 Geometric group theory
20E08 Groups acting on trees
57R30 Foliations in differential topology; geometric theory
37B10 Symbolic dynamics

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