Wu, C. T.; **Hu, W.**; **Chen, J. S.**

A meshfree-enriched finite element method for compressible and near-incompressible elasticity. (English) Zbl 1242.74174

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Summary: In this paper, a two-dimensional displacement-based meshfree-enriched FEM (ME-FEM) is presented for the linear analysis of compressible and near-incompressible planar elasticity. The ME-FEM element is established by injecting a first-order convex meshfree approximation into a low-order finite element with an additional node. The convex meshfree approximation is constructed using the generalized meshfree approximation method and it possesses the Kronecker-delta property on the element boundaries. The gradient matrix of ME-FEM element satisfies the integration constraint for nodal integration and the resultant ME-FEM formulation is shown to pass the constant stress test for the compressible media. The ME-FEM interpolation is an element-wise meshfree interpolation and is proven to be discrete divergence-free in the incompressible limit. To prevent possible pressure oscillation in the near-incompressible problems, an area-weighted strain smoothing scheme incorporated with the divergence-free ME-FEM interpolation is introduced to provide the smoothing on strains and pressure. With this smoothed strain field, the discrete equations are derived based on a modified Hu-Washizu variational principle. Several numerical examples are presented to demonstrate the effectiveness of the proposed method for the compressible and near-incompressible problems.

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References:

[1] Belytschko, Element-free Galerkin methods, International Journal for Numerical Methods in Engineering 37 (2) pp 229– (1994) · Zbl 0796.73077 · doi:10.1002/nme.1620370205

[2] Chen, Reproducing kernel particle methods for large deformation analysis of non-linear structures, Computer Methods in Applied Mechanics and Engineering 139 pp 195– (1996) · Zbl 0918.73330 · doi:10.1016/S0045-7825(96)01083-3

[3] Duarte, A h-p adaptive method using clouds, Computer Methods in Applied Mechanics and Engineering 139 pp 237– (1996) · Zbl 0918.73328 · doi:10.1016/S0045-7825(96)01085-7

[4] Liu, Reproducing kernel particle methods for structural dynamics, International Journal for Numerical Methods in Engineering 38 pp 1655– (1995) · Zbl 0840.73078 · doi:10.1002/nme.1620381005

[5] Melenk, The partition of unity finite element method: basic theory and applications, Computer Methods in Applied Mechanics and Engineering 139 pp 289– (1996) · Zbl 0881.65099 · doi:10.1016/S0045-7825(96)01087-0

[6] Wang, A coupled meshfree/finite element method for automotive crashworthiness simulations, International Journal of Impact Engineering 36 (I10-11) pp 1210– (2009) · doi:10.1016/j.ijimpeng.2009.03.004

[7] Wu, A meshfree procedure for the microscopic analysis of particle-reinforced rubber compounds, Interaction and Multiscale Mechanics 2 pp 147– (2009) · doi:10.12989/imm.2009.2.2.129

[8] Belytschko, A unified stability analysis of meshless particle methods, International Journal for Numerical Methods in Engineering 48 pp 1359– (2000) · Zbl 0972.74078 · doi:10.1002/1097-0207(20000730)48:9<1359::AID-NME829>3.0.CO;2-U

[9] Lancaster, Surfaces generated by moving least squares methods, Mathematics of Computation 37 pp 141– (1981) · Zbl 0469.41005 · doi:10.1090/S0025-5718-1981-0616367-1

[10] Liu, Reproducing kernel particle methods, International Journal for Numerical Methods in Fluids 20 pp 1081– (1995) · Zbl 0881.76072 · doi:10.1002/fld.1650200824

[11] Belytschko, A coupled finite element-element-free Galerkin method, Conputational Mechanics 17 pp 186– (1995) · Zbl 0840.73058 · doi:10.1007/BF00364080

[12] Huerta, Enrichment and coupling of the finite element and meshless methods, International Journal for Numerical Methods in

Engineering 48 (11) pp 1615– (2000) · Zbl 0976.74067 · doi:10.1002/1097-0207(20000820)48:11<1615::AID-NME883>3.0.CO;2-S

[13] Liu, Reproducing kernel element method. Part I: Theoretical formulation, Computer Methods in Applied Mechanics and Engineering 193 pp 933– (2004) · Zbl 1060.74670 · doi:10.1016/j.cma.2003.12.001

[14] Belytschko, Arbitrary discontinuities in finite elements, International Journal for Numerical Methods in Engineering 50 (4) pp 993– (2001) · Zbl 0981.74062 · doi:10.1002/1097-0207(20010210)50:4<993::AID-NME164>3.0.CO;2-M

[15] Babuška, Generalized finite element methods - main ideas, results and perspective, International Journal of Computational Methods 1 (1) pp 67– (2004) · Zbl 1081.65107 · doi:10.1142/S0219876204000083

[16] Babuška, The partition of unity finite element method, International Journal for Numerical Methods in Engineering 40 pp 727– (1997) · doi:10.1002/(SICI)1097-0207(19970228)40:4<727::AID-NME86>3.3.CO;2-E

[17] Millan, Thin shell analysis from scattered points with maximum-entropy approximations, International Journal for Numerical Methods in Engineering 85 pp 723– (2011) · Zbl 1217.74147 · doi:10.1002/nme.2992

[18] Ortiz, Maximum-Entropy meshfree method for compressible and near-incompressible elasticity, Computer Methods in Applied Mechanics and Engineering 199 pp 1859– (2010) · Zbl 1231.74491 · doi:10.1016/j.cma.2010.02.013

[19] Wu, A generalized meshfree approximation for the meshfree analysis of solids, International Journal for Numerical Methods in Engineering 85 pp 693– (2011) · Zbl 1217.74150 · doi:10.1002/nme.2991

[20] Park, On the analysis of dispersion property and stable time step in meshfree method using the generalized meshfree approximation, Finite Element Analysis and Design 47 pp 683– (2011) · doi:10.1016/j.finel.2011.02.001

[21] Beissel, Nodal integration of the element-free Galerkin method, Computer Methods in Applied Mechanics and Engineering 139 pp 49– (1996) · Zbl 0918.73329 · doi:10.1016/S0045-7825(96)01079-1

[22] Chen, A stabilized conforming nodal integration for Galerkin mesh-free methods, International Journal for Numerical Methods in Engineering 50 pp 435– (2001) · Zbl 1011.74081 · doi:10.1002/1097-0207(20010120)50:2<435::AID-NME32>3.0.CO;2-A

[23] Chen, Regularization of material instabilities by meshfree approximations with intrinsic length scales, International Journal for Numerical Methods in Engineering 47 pp 1303– (2000) · Zbl 0987.74079 · doi:10.1002/(SICI)1097-0207(20000310)47:7<1303::AID-NME826>3.0.CO;2-5

[24] Chen, Non-linear version of stabilized conforming nodal integration for Galerkin mesh-free methods, International Journal for Numerical Methods in Engineering 53 pp 2587– (2002) · Zbl 1098.74732 · doi:10.1002/nme.338

[25] Wang, Locking-free stabilized conforming nodal integration for meshfree Mindlin-Reissner plate formulation, Computer Methods in Applied Mechanics and Engineering 193 pp 1065– (2004) · Zbl 1060.74675 · doi:10.1016/j.cma.2003.12.006

[26] Liu, Upper bound solution to elasticity problems: A unique property of the linearly conforming point interpolation method (LC-PIM), International Journal for Numerical Methods in Engineering 74 pp 1128– (2008) · Zbl 1158.74532 · doi:10.1002/nme.2204

[27] Zhao, A linearly conforming radial point interpolation method (LC-RPIM) for shells, Computational Mechanics 43 pp 403– (2009) · Zbl 1162.74512 · doi:10.1007/s00466-008-0313-z

[28] Puso, Meshfree and finite element nodal integration methods, International Journal for Numerical Methods in Engineering 74 pp 416– (2008) · Zbl 1159.74456 · doi:10.1002/nme.2181

[29] Cui, Analysis of plates and shells using an edge-based smoothed finite element method, Computational Mechanics 45 pp 141– (2010) · Zbl 1202.74165 · doi:10.1007/s00466-009-0429-9

[30] Liu, Theoretical aspects of the smoothed finite element method (SFEM), International Journal for Numerical Methods in Engineering 71 pp 902– (2007) · Zbl 1194.74432 · doi:10.1002/nme.1968

[31] Hueck, On the incompressible constraint of the 4-node quadrilateral element, International Journal for Numerical Methods in Engineering 38 pp 3039– (1995) · Zbl 0854.73066 · doi:10.1002/nme.1620381803

[32] Simo, A class of mixed assumed strain methods and the method of incompressible modes, International Journal for Numerical Methods in Engineering 29 pp 1595– (1990) · Zbl 0724.73222 · doi:10.1002/nme.1620290802

[33] Pastor, A mixed displacement-pressure formulation for numerical analysis of plastic failure, Computers and Structures 1 pp 13– (1997) · Zbl 0910.73063 · doi:10.1016/S0045-7949(96)00208-8

[34] Arnold, A stable finite element for the Stokes equations, Calcolo 21 pp 337– (1984) · Zbl 0593.76039 · doi:10.1007/BF02576171

[35] Pierre, Simple C0 approximations for thecomputation of incompressible flows, Computer Methods in Applied Mechanics and Engineering 68 pp 205– (1988) · Zbl 0628.76040 · doi:10.1016/0045-7825(88)90116-8

[36] Lovadina, On the enhanced strain technique for elasticity problems, Computers and Structures 81 pp 777– (2003) · doi:10.1016/S0045-7949(02)00412-1

[37] Arunakirinathar, A stable affine-approximate finite element method, SIAM, Journal on Numerical Analysis 40 pp 180– (2002) · Zbl 1215.74077 · doi:10.1137/S0036142900382442

[38] Huerta, Locking in the incompressible limit for the element-free Galerkin method, International Journal for Numerical Methods in Engineering 51 (11) pp 1361– (2001) · Zbl 1065.74635 · doi:10.1002/nme.213

[39] Vidal, Locking in the incompressible limit: pseudo-divergence-free element free Galerkin, Communications in Numerical Methods in Engineering 19 pp 725– (2003) · Zbl 1112.74545 · doi:10.1002/cnm.631

[40] Chen, An improved reproducing kernel particle method for nearly incompressible finite elasticity, Computer Methods in Applied Mechanics and Engineering 181 pp 117– (2000) · Zbl 0973.74088 · doi:10.1016/S0045-7825(99)00067-5

[41] De, Displacement/pressure mixed interpolation in the method of finite spheres, International Journal for Numerical Methods in Engineering 51 pp 275– (2001) · Zbl 0995.74081 · doi:10.1002/nme.168

[42] Dolbow, Volumetric locking in the element free Galerkin method, International Journal for Numerical Methods in Engineering 46 pp 925– (1999) · Zbl 0967.74079 · doi:10.1002/(SICI)1097-0207(19991030)46:6<925::AID-NME729>3.0.CO;2-Y

[43] Ciarlet, The finite element method for elliptic problems (1978)

[44] Hu, A displacement-based nonlinear finite element formulation using meshfree-enriched triangular elements for the two-dimensional large deformation analysis of elastomers, Finite Element Analysis and Design

[45] Shepard D A two-dimensional interpolation function for irregularly-spaced data Proceedings of the 1968 ACM National Conference 1968 517 524 10.1145/800186.810616

[46] Shephard, Modeling Mesh Generation, and Adaptive Numerical Methods for Partial Differential Equations (1995)

[47] Shannon, A mathematical theory of communication, The Bell Systems Technical Journal 27 pp 379– (1948) · Zbl 1154.94303 · doi:10.1002/j.1538-7305.1948.tb01338.x

[48] Renyi A On measures of entropy and information Proceedings of the 4 th Berkeley Symposium on Mathematical Statistics and Probability 1961 1 547 561

[49] Wilson, Numerical and Computer Models in Structural Mechanics pp 43– (1973) · doi:10.1016/B978-0-12-253250-4.50008-7

[50] César De Sá, New enhanced strain elements for incompressible problems, International Journal for Numerical Methods in Engineering 44 pp 229– (1999) · Zbl 0937.74062 · doi:10.1002/(SICI)1097-0207(19990120)44:2<229::AID-NME503>3.0.CO;2-I

[51] Hughes, The finite element method (2000) · Zbl 1191.74002

[52] Washizu, Vaiational Methods in Elasticity and Plasticity (1982)

[53] Djoko, Conditions for equivalence between the Hu-Washizu and related formulations, and computational behavior in the incompressible limit, Computer Methods in Applied Mechanics and Engineering 195 pp 4161– (2006) · Zbl 1123.74020 · doi:10.1016/j.cma.2005.07.018

[54] Simo, On the variational foundation of assumed strain methods, ASME Journal of Applied Mechanics 53 pp 51– (1986) · Zbl 0592.73019 · doi:10.1115/1.3171737

[55] Timoshenko, Theory of Elasticity (1970)

[56] Hueck, On the stabilization of the rectangular 4-node quadrilateral element, Communications in Numerical Methods in Engineering 10 pp 555– (1994) · Zbl 0804.73058 · doi:10.1002/cnm.1640100707

[57] Wu, Meshfree-enriched simplex elements with strain smoothing for the finite element analysis of compressible and nearly incompressible solids, Computer Methods in Applied Mechanics and Engineering 200 pp 2991– (2011) · Zbl 1230.74201 · doi:10.1016/j.cma.2011.06.013