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On primeness of labeled oriented trees. (English) Zbl 1266.57004

J. Knot Theory Ramifications 21, No. 8, 1250077, 9 p. (2012).

A labeled oriented tree (LOT) is a tree with its n vertices labeled $1, 2, \ldots, n$ in which each edge is given a direction and a unique label in $\{1, 2, \ldots, n\}$. A LOT is called injective if different edges are labeled differently, is called prime if it does not contain a smaller LOT, and is called reduced if: (1) for each edge its own label and the labels of its endvertices are all different, (2) the label of any valence one vertex occurs as the label of some edge, and (3) any two adjacent edges with the same label are neither both into nor both out of their common endvertex. A LOT P gives rise to a group presentation whose generating set is the vertex set and whose defining relations say that the initial endvertex of any edge of P is conjugated to its terminal endvertex by the label of that edge. Let $K(P)$ be the standard 2-complex of the presentation. $K(P)$ can be realized as a spine of the complement of a properly embedded ribbon disc in the 4-ball, see *J. Howie* [Topology 22, 475-485 (1983; Zbl 0524.57002)]. A LOT P is called aspherical if $K(P)$ is aspherical – that is, $\pi_2(K(P)) = 0$. The authors intend to extend the well-known fact that knot complements are aspherical to the case of ribbon disc complements, which is an unsolved problem equivalent to the asphericity problem of LOTs; these problems are related to the Whitehead asphericity conjecture, see *J. Howie* [Trans. Am. Math. Soc. 289, 281–302 (1985; Zbl 0572.57001)].

In a reduced LOT P each relation models a quadrilateral 2-cell in $K(P)$ with labeled, directed sides. $K(P)$ can be represented by a spherical diagram – that is, a quadrangulation of the 2-sphere with different quadrilaterals representing possibly the same 2-cell of $K(P)$. A spherical diagram $C(P)$ is called reducible if some two adjacent quadrilaterals represent the same 2-cell of $K(P)$. A LOT P is said to be diagrammatically reducible (DR) if each spherical diagram over $K(P)$ is reducible. It is known that prime injective LOTs are DR and thereby aspherical. One of the results of the paper is an example of a LOT that is reduced and prime but neither injective, nor DR. This result shows the essentiality of the injectivity condition for the characterization of DR forests established earlier by *G. Huck* and *S. Rosebrock* [Proc. Edinb. Math. Soc., II. Ser. 44, No. 2, 285–294 (2001; Zbl 0983.57003)]. Furthermore, the authors conjecture that all injective LOTs would be aspherical, and prove that under some additional conditions.

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MSC:

57M20 Two-dimensional complexes (manifolds) (MSC2010)
57M05 Fundamental group, presentations, free differential calculus
20F05 Generators, relations, and presentations of groups

Keywords:

Wirtinger presentation; labeled oriented tree; 2-complex; asphericity; diagrammatic reducibility; Whitehead asphericity conjecture

Full Text: DOI

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