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Forbidden graphs for tree-depth. (English) [Zbl 1239.05062](#)

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Summary: For every $k \geq 0$, we define G_k as the class of graphs with tree-depth at most k , i.e. the class containing every graph G admitting a valid colouring $\rho : V(G) \rightarrow 1, \dots, k$ such that every (x, y) -path between two vertices where $\rho(x) = \rho(y)$ contains a vertex z where $\rho(z) > \rho(x)$.

In this paper, we study the set of graphs not belonging in G_k that are minimal with respect to the minor/subgraph/induced subgraph relation (obstructions of G_k). We determine these sets for $k \leq 3$ for each relation and prove a structural lemma for creating obstructions from simpler ones. As a consequence, we obtain a precise characterization of all acyclic obstructions of G_k and we prove that there are exactly $\frac{1}{2}2^{2^{k-1}-k}(1 + 2^{2^{k-1}-k})$. Finally, we prove that each obstruction of G_k has at most $2^{2^{k-1}}$ vertices.

MSC:

[05C15](#) Coloring of graphs and hypergraphs

[05C05](#) Trees

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