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Strong convergence of split-step backward Euler method for stochastic differential equations with non-smooth drift. (English) Zbl 1246.65010

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Under less restrictive assumptions on the drift coefficient f than is customary, the split-step backward Euler method is shown to converge strongly with order $1/2$ to the solution of the Ito stochastic differential equation

$$dX(t) = f(t, X(t))\,dt + g(t, X(r))\,dW(t), \quad 0 \le t \le T, \quad X(0) = X_0.$$

Numerical results are presented that verify that this accuracy is achieved for three examples. Also under even less restrictive assumptions on f , order $1/4$ strong convergence to the solution is proved.

Reviewer: Melvin D. Lax (Long Beach)

MSC:

65C30	Numerical solutions to stochastic differential and integral equations
60H10	Stochastic ordinary differential equations (aspects of stochastic analysis)
60H35	Computational methods for stochastic equations (aspects of stochastic analysis)
34F05	Ordinary differential equations and systems with randomness
65L05	Numerical methods for initial value problems
65L20	Stability and convergence of numerical methods for ordinary differential equations

Keywords:

stochastic differential equations; non-smooth drift; split-step backward Euler method; Euler; Maruyama method; one-sided Lipschitz condition; convergence; Ito stochastic differential equation; numerical results

Full Text: DOI

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